A CASCADED ON-OFF MODEL

OF SESSION TRACES

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CASCADED ON-OFF MODEL

- The cascaded on-off source model has four parameters:
  1. $m =$ the long run mean rate (in bits/sec).
  2. $n =$ order of the cascade (positive integer).
  3. $\lambda =$ basic arrival rate of interruptions (per sec). Level i interruptions arrive according to a Poisson process at rate $2^{i-1}\lambda$ $(i = 1, 2, ..., n)$.
  4. $\mu =$ recovery rate (per sec). It takes an exponential amount of time with mean $1/(2^{i-1}\mu)$ to recover from an interruption of level i $(i = 1, 2, ..., n)$. When the source is recovering from an interruption from level i, other interruptions of level i have no effect.
- Higher the level of interruption, the faster they arrive, and the quicker the source recovers from them.
- Interruption streams from different levels are independent of each other.

THE RATE MODEL

The source produces traffic at rate

$$R_n = m\left(\frac{\lambda+\mu}{\mu}\right)^n$$

whenever it is not recovering from *any* **interruption.**

The expression for R_n above is chosen so that the long run rate at which the source produces traffic is given by m.

MATHEMATICAL MODEL

- Let $X_i(t) = 0$ if the source is recovering from a level i interruption at time t, and 1 otherwise (assuming the source is subjected to interruptions of level i alone).
- $\{X_i(t), t \geq 0\}$ is a Continuous Time Markov Chains (CTMC) on state space $\{0, 1\}$ with rate matrix

$$\begin{bmatrix} -2^{i-1}\mu & 2^{i-1}\mu \\ 2^{i-1}\lambda & -2^{i-1}\lambda \end{bmatrix}.$$

- Let $Y_n(t) = 1$ if the source is transmitting at time t, and 0 otherwise. Then

$$Y_n(t) = \prod_{i=1}^{n} X_i(t), \quad t \geq 0.$$

- Let $Z_n(t)$ be the instantaneous rate at which the source is transmitting traffic at time t. Then

$$Z_n(t) = R_n Y_n(t), \quad t \geq 0.$$

- Show graphs.

ANALYTICAL RESULTS

- $\{Z_n(t), t \geq 0\}$ is an on-off process with iid exponential on times with parameter (i.e., 1/mean) $(2^n - 1)\lambda$, and iid (non-exponential) off times with mean $[(\frac{\lambda+\mu}{\mu})^n - 1]/((2^n - 1)\lambda)$, and long-run mean rate m.
- The mean on time goes to 0 as n goes to infinity.
- The mean off time goes to zero if $\lambda < \mu$, goes to infinity if $\lambda > \mu$ and stays constant $(= 1/\lambda)$ if $\lambda = \mu$ (as $n \to \infty$).
- The ratio of mean off times to the mean on times always goes to infinity (as $n \to \infty$).
- The autocovariance function can be calculated to be

  $$\text{Cov}(Z_n(t), Z_n(0)) = m^2 \left[\prod_{i=1}^{n} \left(1 + \rho e^{-2^{i-1}(\lambda+\mu)t}\right) - 1 \right].$$

  Thus the autocovariance dies off exponentially, dominated by the exponential decay rate of $\lambda + \mu$. Thus, clearly the process will not display long range dependence!

PARAMETER ESTIMATION

The trace data:

1. The peak rate r_{peak} is the true transmission rate of the medium, in bits per second.
2. N = total number of packets in the trace.
3. T_i = start time of the ith packet, in secs ($i = 1, 2, ..., N$).
4. S_i = size of the ith packet, in bits ($i = 1, 2, ..., N$).

The estimation procedure:

1. Estimate the mean on time τ_{on} by
$$\hat{\tau}_{on} = \frac{\Sigma_i S_i}{N r_{peak}}.$$
2. Estimate the mean off time τ_{off} by
$$\hat{\tau}_{off} = \frac{T_N}{N} - \hat{\tau}_{on}.$$
3. For each $n = 1, 2, 3...$ compute
$$\hat{\lambda}_n = \frac{1}{\hat{\tau}_{on}(2^n - 1)},$$
$$\hat{\mu}_n = \frac{\hat{\lambda}_n}{\left[\frac{\hat{\tau}_{off}}{\hat{\tau}_{on}} + 1\right]^{1/n} - 1}.$$

4. Estimate the long run mean rate by

$$\hat{m} = r_{peak} \left(\frac{\hat{\lambda}_n + \hat{\mu}_n}{\hat{\mu}_n} \right)^{-n}.$$

5. There is no unique way to choose n. Try various values, and generate simulated samples. Pick the smallest n that produces "reasonable" looking sample paths.

FUTURE RESEARCH

- Plot the sample paths to see if they "look" reasonable.

- Estimate m, λ and μ in a more sophisticated manner.

- Study the fractal properties of the limit of Z_n as $n \to \infty$.