Introduction to Conservative Cascades

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Basic Construction

Viewpoints: study both of:

- "rate", i.e. density
- "cumulative", i.e. c.d.f

Idea: Apply sequence of "random shocks" at dyadic points.

Conservative Cascade Construction

0. Start Constant: $R_i(t) \equiv M^0$

Relate to top row of EGConsCasc1.ps

1. Split at ½ and for random perturbations M_0^1 and M_1^1 "multiplicatively tweak":

2.

$$R_i(t) = \begin{cases} 2M^0 M_0^1 & t \in [0, \frac{1}{2}] \\ 2M^0 M_1^1 & t \in [\frac{1}{2}, 1] \end{cases}$$

Relate to 2nd row of EGConsCasc1.ps

Note: "conservative" comes from "mass conservation" restriction:

$$M_0^1 + M_1^1 = 1$$

⋮ Continue on dyadic subintervals.

Relate to 3rd row of EGConsCasc1.ps

Simple Conservative Cascade Examples

I. M_i^{ℓ} indep. Uniform(0,1).

a. Rate function looks "very bursty"
Show EGConsCasc2v1d1.ps

b. Dist'n covers wide range of c.d.f.'s
Show EGConsCasc2v2d1.ps

II. M_i^{ℓ} indep. Uniform(0.25,0.75).

a. Rate function "less bursty"
Show EGConsCasc2v1d2.ps

b. Dist'n covers narrower range
Show EGConsCasc2v2d2.ps

Simple Conservative Cascade Examples

III. M_i^{ℓ} indep, $= \begin{cases} .8 & w.p. 1/2 \\ .2 & w.p. 1/2 \end{cases}$

a. Rate function looks "very bursty"

Show EGConsCasc2v1d13.ps

b. Dist'n covers wide range of c.d.f.'s

Show EGConsCasc2v2d13.ps

c. Interesting "fractal" structure.

IV. M_i^{ℓ} indep, $= \begin{cases} .55 & w.p. 1/2 \\ .45 & w.p. 1/2 \end{cases}$

Much less "bursty", and "narrower"

Show EGConsCasc2v1d1.ps and EGConsCasc2v2d1.ps

Cascade Levels

Q: How many levels are needed?

E.g. 1: M_i^ℓ indep. Uniform(0,1).

Show EGConsCasc3v1d1n3.ps … EGConsCasc3v1d1n16.ps

E.g. 2: M_i^ℓ indep, $= \begin{cases} .8 & w.p.\ 1/2 \\ .2 & w.p.\ 1/2 \end{cases}$

Show EGConsCasc3v1d13n3.ps … EGConsCasc3v1d13n16.ps

A: for c.d.f, quite stable for $n = 10 - 16$,
but for rate, continual "divergence"
(steady for discrete)

Conservative Cascade Marginals

0. Constant Rate:

   Show NetDataEG3p1d1r1.ps and NetDataEG3p1d13r1.ps

   All marginals Gaussian

1. M_i^{ℓ} indep. Uniform(0,1):

   show NetDataEG3p1d1r21.ps and NetDataEG3p1d13r21.ps

   Non-Gaussian at smaller scale

2. M_i^{ℓ} indep. Uniform(0.25,0.75)

   show NetDataEG3p1d1r22.ps and NetDataEG3p1d13r22.ps

   Could be Gaussian

3. M_i^{ℓ} indep, $= \begin{cases} .8 & w.p. 1/2 \\ .2 & w.p. 1/2 \end{cases}$

   show NetDataEG3p1d1r25.ps and NetDataEG3p1d13r25.ps

   Non-Gaussian at smaller scale

4. M_i^{ℓ} indep, $= \begin{cases} .55 & w.p. 1/2 \\ .45 & w.p. 1/2 \end{cases}$

   show NetDataEG3p1d1r23.ps and NetDataEG3p1d13r23.ps

   Could be Gaussian

Conservative Cascade Marginals

Summary:

I. Strong "Fractal Effect" (more bursty) $\Rightarrow$
$\Rightarrow$ Non-Gaussian (right skewed)

II. Happens only at smaller scales.

III. Shape of generator dist'n not important, but "spread" is.

IV. Duration dist'n not important.

Inverse Conservative Cascades, I

Motivation: mechanism of TCP/IP Session trace is closer to "horizontal cascade"

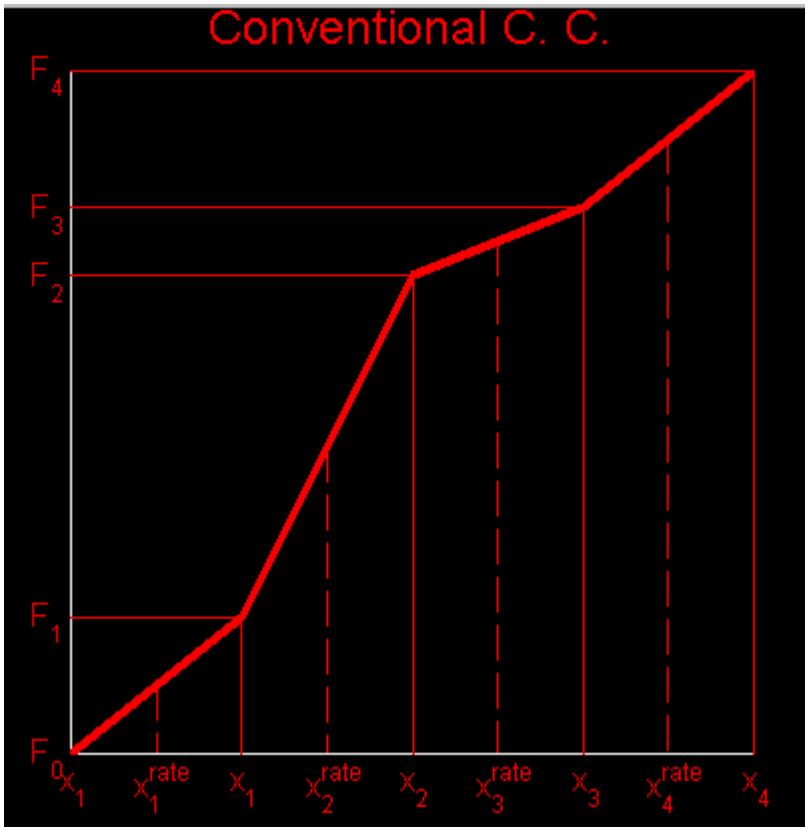
Key concept: "random shocks happen to time, not to rate"

Idea: replace "c.d.f" of original cascade, by "inverse c.d.f."

Rate function: difference this inverse c.d.f.

Inverse Conservative Cascades, II

Relation to Conventional CC:



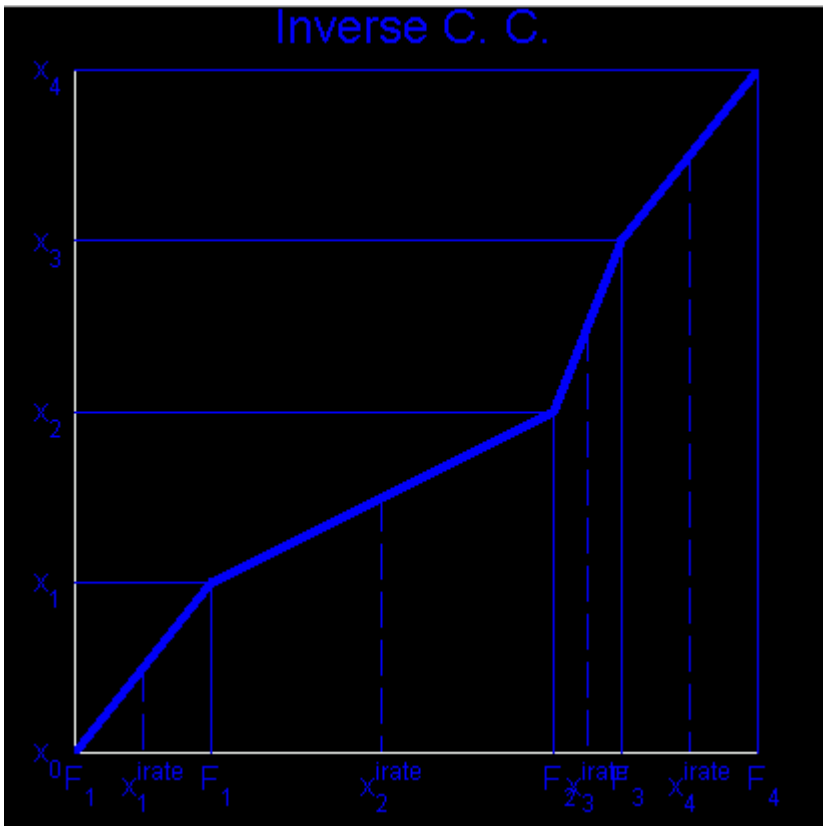
If doesn't show up well, show EGConsCasc1inva.ps

At point $x_k^{rate} = \dfrac{x_k - x_{k-1}}{2}$

display $rate_k = \dfrac{F_k - F_{k-1}}{x_k - x_{k-1}}$

Inverse Conservative Cascades, II (cont.)

Inverse of same



If doesn't show up well, show EGConsCasc1invb.ps

At point $x_k^{irate} = \dfrac{F_k - F_{k-1}}{2}$

display $irate_k = \dfrac{x_k - x_{k-1}}{F_k - F_{k-1}}$

Inverse Conservative Cascades, II (cont.)

Notes:

- $irate_k = \dfrac{1}{rate_k}$
- $\log(irate_k) = -\log(rate_k)$
- so critical difference is in spacings

x_k^{rate} vs. x_k^{irate}

Inverse Conservative Cascades, III

Simple Examples:

e.g. 1 M_i^{ℓ} indep. Uniform(0,1):

Show EGConsCasc2v3d1.ps

- More bursty than conventional CC's

  Relate c.d.f and densities to EGConsCasc2v1d1.ps
- Compelling log rate?
- Has long "buffer-full" stretches?

e.g. 2 M_i^{ℓ} indep. Uniform(0.25,0.75)

Show EGConsCasc2v3d2.ps

- Milder version of above
- Log rate closer to conv. CC

  Compare to EGConsCasc2v1d2.ps
- Have an "extremity parameter"

Inverse Conservative Cascades, IV

e.g. 3 M_i^{ℓ} indep, $= \begin{cases} .8 & w.p. 1/2 \\ .2 & w.p. 1/2 \end{cases}$

Show EGConsCasc2v3d13.ps

- similar properties
- log rate shows "fractal structure"
- log rate much nicer than conv. CC

  Compare to EGConsCasc2v1d13.ps

e.g. 4 M_i^{ℓ} indep, $= \begin{cases} .55 & w.p. 1/2 \\ .45 & w.p. 1/2 \end{cases}$

Show EGConsCasc2v3d11.ps

- similar ideas, and wide family
- log rate very similar to conv. CC

  Compare to EGConsCasc2v1d11.ps
  Again study EGConsCasc1.ps, to see why.

Inverse Conservative Cascades, V

e.g. 5 100 Repl'ed c.d.f. s, Unif(0,1) Gen.

Show EGConsCasc2v4d1.ps and compare to EGConsCasc2v2d1.ps

- "axis flip" of same for conv. CC
- qualitatively similar
- bigger "slow time periods"
- main difference in "discretization"?

e.g. 6 Level-wise Unif(0,1) Gen.

Show EGConsCasc3v2d1n3.ps … EGConsCasc3v2d1n16.ps

- "off periods" depend on level
- and eventually get traffic

e.g. 7 Level-wise Bernoulli Gen.

Show EGConsCasc3v2d13n3.ps … EGConsCasc3v2d13n16.ps

- development of "fractal structures"
- "off periods" diminish in log rate

Problem Solved?

Riedi & Willinger (1998): Sec. 1.3.3:

- …this work leaves open the "big" question "Why are packets within individual TCP connections distributed in accordance with a conservative cascade construction?"…

- Clearly progress on these problems will require a close collaboration with networking experts.

- There exists great potential for coming up with intuitively appealing, conceptually simple and mathematically rigorous statements as to the causes and effects of multifractals in data networking.

CC Generator Distributions

Idea: "solve CC generation eq'ns for M_k^j"

Show EGConsCasc1.ps

Recall, at CC level j, mass assigned to:

Bin k (even) is $M^0 M_{k'}^1 M_{k''}^2 \cdots M_{k^+}^{j-1} M_k^j$

Bin $k+1$ is $M^0 M_{k'}^1 M_{k''}^2 \cdots M_{k^+}^{j-1} M_{k+1}^j$

Where $M_k^j + M_{k+1}^j = 1.$

Ratio of masses gives $R = \dfrac{M_k^j}{M_{k+1}^j}.$

So can iteratively solve for all of the M_k^j

CC Generator Decomposition

Solution: $M_k^j = \frac{R}{R+1}, \quad M_{k+1}^j = \frac{1}{R+1}$

"nonlinear", but "well behaved"

Duality: as for Fourier or Wavelet trans.

given time series, get CC Gen. Decomp.

given CC Gen. Decomp. can get series

Viewpoint: M_k^j reveals "structure in data"

"Good case": Generator Dist'ns stationary and independent across levels j

CC Gen. Decomp. Example 1

Discretely generated CCs:

$$M_i^{\ell} \text{ indep, } = \begin{cases} .8 & w.p. 1/2 \\ .2 & w.p. 1/2 \end{cases}$$

Show EGConsCasc5v1d13.ps and EGConsCasc5v1d13j.ps

- all gen's at 0.2 and 0.8 (as expected)
- means feel "Binomial variability"

CC Gen. Decomp. Example 2

UNC Main Link data (major aggregation)

View 1: Combine M_k^j across levels j

Show UncLinkdata/ UncLinkData2p31d1.ps and UncLinkData2p32d1.ps

Notes:

1. QQ and SiCon both suggest "scale mixture of normals":
$$\sum_i \omega_i N\left(0.5, \sigma_i^2\right)$$

2. Similar for "counts" and "sizes"

3. "counts" have more "fine scale features"

CC Gen. Decomp. Example 2 (cont.)

View 2: Level wise

Show UncLinkdata/ UncLinkData2p33d1.ps and UncLinkData2p34d1.ps

Notes:

1. Each looks Gaussian
2. Gaussian time series $\Rightarrow$ Gaussian CC Gen. Decomp.???
3. Means constant, and = $1/2$, across levels
4. Big change in s.d. across levels
5. Increasing quadratic structure to s.d.s???
6. Bigger s.d. for Packet Sizes

CC Gen. Decomp. Example 3

CC Generated Series:

Unif(0,1):

Show EGConsCasc5v1d1.ps, EGConsCasc5v1d1j.ps and EGConsCasc5v1d1s.ps

- dist'ns "look uniform" (expected)

- study dependence across levels via:

$$\left\{\left(\left|M_{2k}^{j}-0.5\right|,\left|M_{k}^{j-1}-0.5\right|\right): k=0,\ldots,2^{j-1}-1\right\}$$

- seems independent across levels

Unif(0.25,0.75)

Show EGConsCasc5v1d2.ps, EGConsCasc5v1d2j.ps and EGConsCasc5v1d2s.ps

- similar lessons, but "compressed" as expected

CC Gen. Decomp. Example 3 (cont.)

Unif(0,1) inverse:

Show EGConsCasc5v1d6.ps, EGConsCasc5v1d6j.ps and EGConsCasc5v1d6s.ps

- Non-Gaussian shape
- "Pileup" near 0.5, at finer levels
- Contrast s.d. behavior & Link Data
  Again show UncLinkData\ UncLinkData2p33d1.ps
- Jitter Plot $\Rightarrow$ mix of 0.5 and Unif.?
- Hence do "External kde" & measure "% outside"
- Some dependence? (need stat'l sig.)
- Caused by "flat spots"?
- Lesson: CC not good model here?

CC Gen. Decomp. Example 4

(Explain behavior with more examples)

"Step" rates:

1 Step:

Show EGConsCasc5v1d33.ps and EGConsCasc5v1d33j.ps

- vert. & horiz. rates much different
- more spread at coarser levels
- all but two at 0.5 (each level)
- have 0.5 at "linear places" in c.d.f.

CC Gen. Decomp. Example 4 (cont.)

Inverse 1 Step:

Show EGConsCasc5v1d38.ps and EGConsCasc5v1d38j.ps

- rates are swapped
- interpolation singularity
- other lessons are same

5 Step:

Show EGConsCasc5v1d34.ps and EGConsCasc5v1d34j.ps

- similar ideas
- few more $\neq$ 0.5

CC Gen. Decomp. Example 5

"Smooth deterministic signal"

5 cycle sin:

Show EGConsCasc5v1d31.ps, EGConsCasc5v1d31j.ps and EGConsCasc5v1d31s.ps

- interesting rate patterns
- finer scale $\Rightarrow$ decr. s.d. ($\approx$ linear)
- correlations $\rightarrow$ 1
- scatterplot $\Rightarrow$ "dynamical system"?
- bad stationarity, bad dependence, conclude CC is a poor model

CC Gen. Decomp. Example 6

"White Noise":

Generate n Unif(0,1)s & bin (1024 bins)

$n = 10{,}000$ (more data than bins)

Show EGConsCasc5v1d23.ps, EGConsCasc5v1d23j.ps and EGConsCasc5v1d23s.ps

- rate functions like CC?

- finer scale $\Rightarrow$ increasing s.d.

  compare to Link Data

  Again show UncLinkData\UncLinkData2p33d1.ps

- Gaussian dist'ns

- indep. across scales

CC Gen. Decomp. Example 6 (cont.)

$n = 100$ (more bins than data)

Show EGConsCasc5v1d21.ps, EGConsCasc5v1d21j.ps and EGConsCasc5v1d21s.ps

- rate functions often ≈ 0
- s.d. increases then decreases?
- reason (from jitter plot):
  at fine scales, c.d.f. is mostly linear
  at coarse scales, similar to above
- gen's mostly at edges of scatterplot
- all are "too discrete" effects, which diminishes at coarser scales

Open Problems

I. Prob. Limit Theorems for ICC model?

II. Connections between CC and ICC?

III. Mathematics for CC Generator Dist'ns

IV. Strategy for papers?