Multi Fractal Analysis

Led by Steve Marron, 5/5/00, 5/12/00

Main idea: c.d.f.s of "bursty" processes will have:

Show ConsCasc\EGConsCasc2v2d1.ps

- more steep bits "than usual"

- more flat bits "than usual"

How to quantify this?

Fractal Index

Index “steepness of c.d.f.”, using α

On scale of:

$\alpha < 1 \quad \Leftrightarrow$ “really steep”

$\alpha = 1 \quad \Leftrightarrow$ “steep-flat at random”

$\alpha > 1 \quad \Leftrightarrow$ “really (close to) flat”

$\alpha = \infty \quad \Leftrightarrow$ ‘’exactly flat”

Multifractal Spectrum

Idea: for each α,

How many bits have steepness α?

Math's get complicated (fractal dim'n),

but think "relatively".

Toy Example

E.g. 1: White Noise

Show mfeg21p1d1.ps

MultiFractal Spectrum:

- biggest for $\alpha \approx 1$
- “falls away” for larger α
- “falls away” for smaller α
- sensible for “usual random c.d.f.”

Under The Hood

1. Start with partition function

$$S^{(k)}(q) = \sum_i \left| X_i^{(k)} \right|^q$$

2. Find slopes , $\tau(q)$ on log-log scale by fitting a line (to "straight" part)

   a. need to "truncate" (but where?)

   b. use "robust repeated median", to "make automatic"

Show MedLinRegT20.ps and mfeg21p2d1.ps

3. Take Legendre transform:

$$MFS(\alpha) = \min_q \left(q\alpha - \tau(q) \right)$$

More Toy Examples

E.g. 2: Cons. Casc., Unif(0,1) Gen'd

Show ConsCasc\EGConsCasc1.ps & ConsCasc\EGConsCasc2v1d1.ps and mfeg21p2d6.ps

- MFS much "broader"

- i.e. wider array of "flats and steeps"

- MFS hits 1 at $\alpha \approx 1.37$

- i.e. "most common" is that flat bit

- robust linear fit works

More Toy Examples

E.g. 3: C. C., Unif(0.25,0.75) Gen'd

Show mfeg21p2d7.ps

- MFS "less spread"
- MFS "more spread" than for white noise.

E.g. 4: C. C., Bernoulli(0.2-0.8) Gen'd

Show mfeg21p2d8.ps

- MFS again "quite spread"
- MFS max'd at large $\alpha \approx 1.31$

More Toy Examples

E.g. 5: C. C., Ber(0.45-0.55) Gen'd

Show mfeg21p2d9.ps

- MFS very similar to white noise

E.g. 6: Fractional Brownian Motions
Hurst Parameter H = 0.5, 0.7, 0.9, 1

Show mfeg21p2d2.ps, mfeg21p2d3.ps, mfeg21p2d4.ps, and mfeg21p2d5.ps

- MFS gets "slight larger on right"

- MFS very similar to white noise

- Really different from CC's

UNC Main Link Data

Packet Counts per Unit Time:
(on scale m = 0.003 sec)

Show UncLinkData\UncLinkData2p41d1.ps and UncLinkData2p41d2.ps

- MFS consistent with Fractional Brownian Motion

- Predicted because of "large aggregation"

Bytes per Unit Time:

Show UncLinkData\UncLinkData2p42d1.ps and UncLinkData2p42d2.ps

- similar lessons

UNC Session Data

Show UNCSessionData/ CombineSessionData1p1.pdf

Time Interpolated View:

Show ConsCasc/EGConsCasc1int.ps and UNCSessionData/CombineSessionData1p14.pdf

- MFS suggests multi-fractal struct.
- MFS drops on right: few flats
- MFS high on left: many steeps
- max $\alpha \approx 1$ driven by “regularity of flat bits”?
- max MFS << 1, because many “truly flat” bits
- Robust linear fits work well? (Rolf is happy…)

UNC Session Data

Size Interpolated View

Show ConsCasc/EGConsCasc1int.ps and UNCSessionData/CombineSessionData1p15.pdf

- general trend in MFS is reversed (theoretically predicted)
- is MFS “looks more quadratic”, related to “inverse CC gives better visual fit”???
- max α is consistently >> 1.
- max MFS = 1 (expected)
- Both time and size interpolated contain “useful information”
- Robust linear fits work well?

Big Picture Summary of MFA

1. Main Link Data

   - “Monofractal”   (~ WN, FBM)

   - Expected from aggregation

2. Session Data

   - “Multfractal”, mult’ive not additive

   - Size Interp. & Time Interp. are both useful

3. Look at data “in between”???