Ideas from: Riedi and Willinger (1999)

Riedi, R. H. and Willinger, W. (1999) “Toward an improved understanding of network traffic dynamics”, in *Self-similar network traffic and performance evaluation*, Wiley.

Discussed on Feb. 4 & 25, 2000,

led by Steve Marron, UNC, Statistics

Main points:

- overview of series of models
- good “historical” perspective
- useful motivation for decisions
- clear discussion of assumptions and strengths/weaknesses

Context: Internet traffic

Basic Concept: series of "sessions", where packets interchanged between 2 points.

Model for traffic at intermediate node:

$$X = \sum_i X_i$$

Show NetDataEG1d1r1p1.ps (sorted by start time)
and NetDataEG1d1r1p2.ps (randomly ordered start time, "better" view?)

where:

X = "total traffic rate"

i = index of session

X_i = "session traffic rate"

Time Scalings: indexed by m

e.g. $X^{(m)}(k)$ is:

- number of packet arrivals per unit time
- total size of packets per unit time

during the $k-$th time interval,
of length m.

$X_i^{(m)}(k)$ is $i-$th connection version

Multiscale approach:
study several “levels of resolution”, m

Section 1.2.1: Additive $\Rightarrow$ Gaussian Dist.

For each fixed k:

$$X^{(m)}(k) = \sum_i X_i^{(m)}(k),$$

thus tend toward Gaussian by:

- "law of averages"
- "Central Limit Theorem"
- $\sqrt{n}$ rate of convergence

But: this assumes

- "mixing" (e.g. independence)
- 2nd moments

Bases for Models:

S_i - Start time of $i-$th session

D_i - Duration time of $i-$th session

point these out in NetDataEG1d1r1p2.ps

$R_i(t)$ - Rate function for $i-$th session

(maybe time local)

Show constant rate in NetDataEG1d1r1p4.ps

Important concepts:

- Model S_i and D_i as random
- Dist'ns drive prob's for $X^{(m)}$
- "Technical" assumptions crucial

“Classical Assumptions”

(work horses for telephone system –
standard queuing theory)

$\{S_i : i = 1,...\}$ is homogeneous Poisson

(have tools to check this, worthwhile?)
(Cleveland: much different model)

D_i are indep. Exp. (“mean” τ, “rate” $\frac{1}{\tau}$)

Again show NetDataEG1d1r1p2.ps

$R_i(t)$ is constant

Again show NetDataEG1d1r1p4.ps

Section 1.2.2: Heavy Tails $\Rightarrow$ Self-similar

Key: distribution of "durations" D_i

Tail Index α:

$$P\{D_i > t\} = 1 - F_D(t) \approx c \cdot t^{-\alpha}, \quad \text{as} \;\; t \to \infty.$$

Interesting case: $1 < \alpha < 2$:

$$E\{D_i\} < \infty,$$

$$\text{var}\{D_i\} = \infty.$$

i.e. get occasional very large values.

Show NetDataEG1d1r1p2.ps and NetDataEG1d13r1p2.ps, comment: need "longer time scale", so show NetDataEG1d1r1p3.ps, NetDataEG1d11r1p3.ps, NetDataEG1d12r1p3.ps and NetDataEG1d13r1p3.ps

(have tools to check this, worthwhile?)

Claimed Consequence:

$1 < \alpha < 2 \quad \Rightarrow \quad$ long range dependence

For autocorrelation (at all scales m)

$$r^{(m)}(k) = \rho\left(X^{(m)}(\ell), X^{(m)}(\ell + k)\right)$$

have "slow convergence towards indep":

$$r^{(m)}(k) \approx c \cdot k^{2H-2} \quad \text{as} \quad k \to \infty.$$

(have tools to check this, worthwhile?)

H is "Hurst param" = Self-similarity param,
$H = (3 - \alpha)/2$.

Note: $1 < \alpha < 2 \quad \Rightarrow \quad 1/2 < H < 1.$

Contrast: For usual ARMA time series:

$$r^{(m)}(k) \approx c \cdot \phi^k \quad \text{as} \quad k \to \infty.$$

(exponential, not polynomial)

NetDataEG1d1r1p5.ps, (log scale and green line show exponential fit for lags 1-40)
NetDataEG1d11r1p5.ps, (exponential fit still reasonable)
NetDataEG1d1r12p5.ps, (now have slight, but systematic curvature away from line)
NetDataEG1d13r1p5.ps (slightly stronger curvature)

Claimed Consequence: "Self-similarity"

For time scales m_1 and m_2

$$m_1^{1-H} X^{(m_1)} \quad \text{“looks like”} \quad m_2^{1-H} X^{(m_2)}$$

Compare: Central Limit Theorem:

$$n_1^{1/2} \overline{X}_{n_1} \ \sim \ n_2^{1/2} \overline{X}_{n_2} \ \sim \ N(0, \sigma^2)$$

i.e. self-similarity parameter, $H = 1/2$

Self similarity summary:

(i) intrinsically additive

(ii) driven by session properties

(iii) "averages out" packet flow details

(iv) holds for wide range of setups

(v) assumes approx. constant rates, $R_i(t)$

Section 1.2.3: Self-similar Gaussian Models

For $X^{(m)}$ Gaussian,

Consider Gaussian and Q-Q parts of NetDataEG1d1r1p5.ps, NetDataEG1d11r1p5.ps, NetDataEG1d1r12p5.ps, and NetDataEG1d13r1p5.ps

(i.e. "marginals" Gaussian; i.e. for each k)

$$m^{1-H} X^{(m)} \quad \text{"looks like"} \quad X$$

NetDataEG1d1r1p6.ps, tried to show this but, sd's go wrong way! "Local dependence"?
NetDataEG1d1r1p7.ps, "decimated", intead of zoomed, note finer scales still "all look the same" (even the same realization!). Try much shorter durations.
NetDataEG1d1r1p8.ps, now s.d. goes down "like sqrt(m) ~ 2
NetDataEG1d11r1p8.ps, similar decrease of s.d.
NetDataEG1d12r1p8.ps, s.d. goes down slower?
NetDataEG1d13r1p8.ps, clearly slower decrease of sd. (decimated histo's were similar)
NetDataEG2p2.ps, shows rate of decrease, and compares with theoretical 1-H, note about %20 under, likely due to dependence

where X is "Fractional Gaussian Noise",

which has autocorrelation:

$$r(k) = \frac{1}{2}\left[(k+1)^{2H} - 2k^{2H} + (k-1)^{2H}\right]$$

(have tools to check this, worthwhile?)

Assumptions:

1. Summed over many independent connections (gives Gaussian).

2. Single connection flow control negligible (OK for large m?)

3. Time scales coincide with scaling.

Claim: OK, at “backbone”, where have high levels of aggregation.

Section 1.2.4: Toward non-Gaussian Models

Key idea: Heavy tailed packet rates, R_i

Again show constant rate in NetDataEG1d1r1p4.ps, and then show Cox process rates in NetDataEG1d1r11p4.ps

Model $R_i(k)$ as random, with (heavy) tail index $\beta \in (1,2)$.

Then change Hurst Index to:

$$H = \frac{\beta - \alpha + 1}{\beta} = 1 - \frac{\alpha - 1}{\beta}$$

Note 1:

$$\beta = 2 \quad \Rightarrow \quad H = \frac{3 - \alpha}{2} \quad \text{(as before)}$$

Note 2:

Heavier Tail (smaller β) $\Rightarrow$

$\Rightarrow$ larger $\frac{\alpha-1}{\beta}$

$\Rightarrow$ smaller $1-\frac{\alpha-1}{\beta}$

$\Rightarrow$ smaller H

$\Rightarrow$ less long range dependence?

Possible interpretation: "higher vert'l noise" swamps horiz'l dependence????

(worth a deeper look?)

Section 1.3: "Small time" scaling behavior

Approach: model (random) $R_i(t)$ by
"conservative cascades"

Idea: Apply sequence of "random shocks"
at dyadic points.

Conservative Cascade Construction

0. Start Constant: $R_i(t) \equiv M^0$

Relate to top row of EGConsCasc1.ps

1. Split at ½ and for random perturbations M_0^1 and M_1^1 "multiplicatively tweak":
2.

$$R_i(t) = \begin{cases} 2M^0 M_0^1 & t \in [0, \frac{1}{2}] \\ 2M^0 M_1^1 & t \in [\frac{1}{2}, 1] \end{cases}$$

Relate to 2nd row of EGConsCasc1.ps

Note: "conservative" comes from "mass conservation" restriction:

$$M_0^1 + M_1^1 = 1$$

⋮ Continue on dyadic subintervals.

Relate to 3rd row of EGConsCasc1.ps

Simple Conservative Cascade Examples

I. M_i^ℓ indep. Uniform(0,1).

a. Rate function looks "very bursty"
Show EGConsCasc2v1d1.ps

b. Dist'n covers wide range of c.d.f.'s
Show EGConsCasc2v2d1.ps

II. M_i^ℓ indep. Uniform(0.25,0.75).

a. Rate function "less bursty"
Show EGConsCasc2v1d2.ps

b. Dist'n covers narrower range
Show EGConsCasc2v2d2.ps

Simple Conservative Cascade Examples

III. M_i^ℓ indep, $= \begin{cases} .8 & w.p. 1/2 \\ .2 & w.p. 1/2 \end{cases}$

a. Rate function looks "very bursty"

Show EGConsCasc2v1d13.ps

b. Dist'n covers wide range of c.d.f.'s

Show EGConsCasc2v2d13.ps

c. Interesting "fractal" structure.

IV. M_i^ℓ indep, $= \begin{cases} .55 & w.p. 1/2 \\ .45 & w.p. 1/2 \end{cases}$

Much less "bursty", and "narrower"

Show EGConsCasc2v1d1.ps and EGConsCasc2v2d1.ps

Conservative Cascade Marginals

0. Constant Rate:

   Show NetDataEG3p1d1r1.ps and NetDataEG3p1d13r1.ps

   All marginals Gaussian

1. M_i^ℓ indep. Uniform(0,1):

   show NetDataEG3p1d1r21.ps and NetDataEG3p1d13r21.ps

   Non-Gaussian at smaller scale

2. M_i^ℓ indep. Uniform(0.25,0.75)

   show NetDataEG3p1d1r22.ps and NetDataEG3p1d13r22.ps

   Could be Gaussian

3. M_i^ℓ indep, $= \begin{cases} .8 & w.p. 1/2 \\ .2 & w.p. 1/2 \end{cases}$

   show NetDataEG3p1d1r25.ps and NetDataEG3p1d13r25.ps

   Non-Gaussian at smaller scale

4. M_i^ℓ indep, $= \begin{cases} .55 & w.p. 1/2 \\ .45 & w.p. 1/2 \end{cases}$

   show NetDataEG3p1d1r23.ps and NetDataEG3p1d13r23.ps

   Could be Gaussian

Conservative Cascade Marginals

Summary:

I. Strong "Fractal Effect" (more bursty) $\Rightarrow$
$\Rightarrow$ Non-Gaussian (right skewed)

II. Happens only at smaller scales.

III. Shape of generator dist'n not important, but "spread" is.

IV. Duration dist'n not important.

Multi Fractal Analysis

Idea: provide "spectrum" of fractal components???

Summary of queries about what to check:

1. $\{S_i : i = 1,...\}$ is homogeneous Poisson

2. Tail index of “durations” D_i

3. Hurst index of autocorrelation $r^{(m)}(k)$.

4. Gaussian dist’n and Fractional Gaussian form of auto-corr.

5. Deeper look at “heavy tailed rates swamps long range dependence”?