Linear Algebra Review, (cont.)

Eigenvalue Decomposition:

For a (symmetric) square matrix $X_{d \times d}$

Find a diagonal matrix $D = \begin{pmatrix} \lambda_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_d \end{pmatrix}$

And an orthonormal matrix $B_{d \times d}$ (i.e. $B^t \cdot B = B \cdot B^t = I_{d \times d}$)

So that: $X \cdot B = B \cdot D$, i.e. $X = B \cdot D \cdot B^t$

Linear Algebra Review, (cont.)

Eigenvalue Decomposition (cont.):

Relation to Singular Value Decomposition (looks similar?):

- Eigenvalue decomposition "harder"
- Since needs $U = V$
- Price is eigenvalue decomp'n is generally complex
- Except for X square and symmetric
  - Then eigenvalue decomposition is real valued
  - Thus it is the sing'r value decomp. with $U = V = B$

Linear Algebra Review, (cont.)

Computation of Singular Value and Eigenvalue Decompositions:

- Details too complex to spend time here
- A “primitive” of good software packages
- Eigenvalues $\lambda_1, \ldots, \lambda_d$ are unique
- Columns of $B = \begin{pmatrix} \underset{\rightarrow}{v_1} & \cdots & v_d \end{pmatrix}$ are called “eigenvectors”
- Eigenvectors are “λ-stretched” by X:

$$X \cdot \underset{\rightarrow}{v_i} = \lambda_i \cdot \underset{\rightarrow}{v_i}$$

Linear Algebra Review, (cont.)

Eigenvalue Decomposition solves matrix problems:

- Inversion: $$X^{-1} = B \cdot \begin{pmatrix} \lambda_1^{-1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_d^{-1} \end{pmatrix} \cdot B^t$$

- Square Root: $$X^{1/2} = B \cdot \begin{pmatrix} \lambda_1^{1/2} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_d^{1/2} \end{pmatrix} \cdot B^t$$

- $rank(X) = \#\{\lambda_i : \lambda_i \neq 0\}$

- X is positive (nonnegative, i.e. semi) definite $\Leftrightarrow$

  $\Leftrightarrow$ all $\lambda_i > (\geq) 0$

[go to next part]