PCA dual problem (cont.)

Now suppose know sol'n to dual problem, i.e. know B^* and D^*

How do we find B and D?

A heuristic approach:

i. want B so that

$$D = B^t \hat{\Sigma} B = B^t \tilde{X}\tilde{X}^t B$$

PCA dual problem (cont.)

ii. choose B to introduce form $\tilde{X}^t\tilde{X} = \Sigma^*$,
i.e. $B = \tilde{X}C$ (for some C), then

$$D = C^t\tilde{X}^t\left(\tilde{X}\tilde{X}^t\right)\tilde{X}C = C^t\left(\tilde{X}^t\tilde{X}\right)\left(\tilde{X}^t\tilde{X}\right)C = C^t\Sigma^*\Sigma^*C$$

iii. choose C to relate to $\Sigma^* = B^*D^*B^{*t}$, i.e. $B^{*t}\Sigma^*B^* = D^*$
i.e. $C = B^*R$ (for some R), then

$$D = C^t\Sigma^*\left(B^*B^{*t}\right)\Sigma^*C = \left(R^tB^{*t}\right)\Sigma^*B^*B^{*t}\Sigma^*\left(B^*R\right)$$

$$D = R^t\left(B^{*t}\Sigma^*B^*\right)\left(B^{*t}\Sigma^*B^*\right)R = R^tD^*D^*R$$

PCA dual problem (cont.)

iv. Choose R to "preserve energy",

i.e. "make B orthonormal",

i.e. "make B a rotation matrix",

i.e. choose $R = \left(D^*\right)^{-1/2}$, then

$D = D^*$, i.e. same (nonzero) eigenvalues!

PCA dual problem (cont.)

Heuristic summary: Want $B = \tilde{X}C = \tilde{X}\left(B^{*}R\right) = \tilde{X}B^{*}\left(D^{*}\right)^{-1/2}$

Technical problems:

- dimensions wrong: $B_{d\times d}$, $\tilde{X}_{d\times n}$, $B^{*}_{n\times n}$, $D^{*}_{n\times n}$
- $D^{*}_{n\times n}$ not full rank?, thus:
    - root inverse doesn't exist?
    - B is not a basis matrix

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