Statistics 31, Section 3, Midterm II, Solution
Tuesday, November 14, 2000

Name: ____

Pledge: I have neither given nor received aid on this examination.

Signature: ____

Instructions: Do <u>not</u> do any actual numerical calculations. Answers in a form that you would type into an Excel field, such as "=28*SQRT(82)^2", with a *working* answer, are expected).
[points per part]

1. A company makes 50% of its cars at Factory A, 30% at Factory B and the rest at Factory C. Factory A produces 10% lemons, Factory B produces 15% lemons and Factory C produces 5% lemons. A car is chosen at random. What is the probability that:

 a. It is a lemon?
[10]

P{A} = 0.50, P{B} = 0.30, P{C} = 0.20

P{L|A} = 0.10, P{L|B} = 0.15, P{L|C} = 0.05

P{L} = P{(L and A) or (L and B) or(L and C)} =
= P{L and A} + P{L and B} + P{L and C} =
= P{L|A} P{A} + P{L|B} P{BP} + P{L|C} P{C} =
= 0.10*0.50 + 0.15*0.30 + 0.05*0.20

 b. It came from Factory B if it is a lemon?
[10]

P{B|L} = P{B and L} / P{L} = …

Or Bayes Rule:
P{B|L} = P{L|B} P{B} / (P{L|A} P{A} + P{L|B} P{B} + P{L|C} P{C})
= (0.15*0.30) / (0.10*0.50 + 0.15*0.30 + 0.05*0.20)

[20]

2. The weights of a random sample of 25 runners averaged 60 kg. Suppose that the standard deviation of the population is known to be 10 kg.

   a. What is $\sigma_{\bar{X}}$, the standard deviation of the sample average $\bar{X}$?

[5]

sigma / sqrt(n) = 10 / sqrt(25)

   b. Find the 99% margin of error for estimating the population mean μ using $\bar{X}$.

[5]

=confidence(0.01,10,25)

   c. Give a 90% confidence interval for μ.

[5]

left end: =60-confidence(0.1,10,25)
right end: =60+confidence(0.1,10,25)

   d. Exactly how would the confidence interval in (c) change if the sample average were based on a random sample of 100 runners?

[5]

same center, but length would decrease by factor of sqrt(4) = 2

   e. How large a sample would be required to estimate μ within ± 0.1 kg with 95% confidence?

[5]

=(10*NORMINV(0.975,0,1)/0.1)^2

[25]

3. A household is called prosperous if its income exceeds $75,000, and called educated if the householder completed college. 20% of all households are prosperous, 30% are educated, and 19% are prosperous and educated. If a household is chosen at random:

a. What is the probability that it either is educated, or else is prosperous?

[5]

P{Ed. or Pros.} = P{Ed.} + P{Pros.} –P{Ed. and Pros.} =
= 0.30 + 0.20 –0.19

b. What is the probability that it is educated given that it is prosperous?

[5]

P{Ed.|Pros.} = P{Ed. and Pros.} / P{Pros.} =
= 0.19 / 0.20

c. Is the event that it is educated independent of the event that it is prosperous? Why or why not?

[5]

No, P{Ed.|Pros.} is not equal to P{Ed.}

4. A box label claims that on average boxes contain 40oz. A random sample of 12 boxes shows an average of 39oz., with s = 2.2. To see if we should dispute the claim, consider the hypotheses:

$$H_+: \quad \mu > 40 \qquad H_0: \quad \mu = 40 \qquad H_-: \quad \mu < 40$$

a. Find the p-value to assess the strength of the evidence in favor of H_+.

[10]

p-val = P{what was seen or more conclusive | H0} =
= P{Xbar > 39 | mu = 40} =
= 1 - NORMDIST(39,40,2.2/SQRT(12),TRUE)

b. If the p-value to test H_-: were equal to 0.0613, interpret the results from both the "yes-no" and the "gray level" viewpoints.

[5]

yes – no: no strong evidence at level 0.05
gray level: evidence is somewhat strong in the direction of the label being wrong. [30]

5. According to government data, 15% of employed men have never been married.

a. If 12 employed men are selected at random, what is the probability that at least 10 have never been married?

[5]

let X = # is sample never been married. X ~ Bi(12,0.15)

P{X >= 10} = 1 – P{X < 10} = 1 – P{X <= 9}
= 1 – BINOMDIST(9,12,0.15,TRUE)

b. If 12 employed men are selected at random, what is the probability that less than 4 have been married?

[5]

Use X as above, P{less than 4 have been} = P{more than 8 never been} = P{X > 8} =
1 – P{X<=8} = 1 – BINOMDIST(8,12,0.15,TRUE)

or let X = # have been married, X ~ Bi(12,0.85)
P{X < 4} = P{X <= 3} = BINOMDIST(3,12,0.85,TRUE)

c. 12 employed men are selected at random, what is the mean number that have never been married?

[5]

mean = n p = 12 * 0.15

d. Let X denote number the number who have never been married, in a random sample of 12 employed men. What is the standard deviation of X ?

[5]

s.d. = sqrt(n p (1 – p)) = SQRT(12 * 0.15 * (1 – 0.15))

e. If 1200 employed men are selected at random, what is the probability that at least 100 have never been married?

[5]

Note: BINOMDIST gives an error message, so must use normal approx:
X ~N(np,sqrt(np(1-p))
P{X >= 100) = 1 – P{X <= 100}
= 1 – NORMDIST(100,12*0.15,SQRT(12*0.15*(1-0.15)),TRUE) [25]