Statistics 23, Midterm 1
Tuesday, February 15, 1994

Name:____

Pledge: I have neither given nor received aid on this examination.

Signature:____

Instructions: Show all work. Note that a binomial table is supplied.

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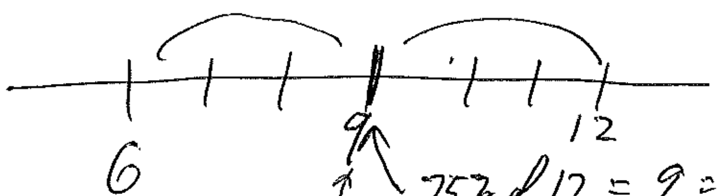
1. A ketchup company's marketing department used a telephone survey of 12 randomly selected households, and found that 6 of them were using their extra spicy ketchup. Last year a much more extensive showed that the market share for the extra spicy ketchup was 75%. Is it safe to conclude that their market share for extra spicy ketchup is different this year?

let p = proportion using ex. spicy $\quad H_0: p = .75 \quad H_1: p \neq .75$

X = # in 12 using ex. spicy $\sim Bi(12, p)$ 1-sided? (-10)



75% of 12 = 9 = "most likely"

center at 8? (-3) center at 6? (-5)

OK, but not? (-6)

p-value = $P[X = 6$ or m.c. $|$ Bdry$] = P[X \leq 6$ or $X \geq 12 \,|\, p = .75]$

$p = .5$? (-5)

P[X=6]? (-10)

$= P[n-X \geq \overset{6}{n-6}$ or $n-X \leq \overset{0}{n-12} \,|\, p = .75]$

$n-X \sim Bi(12, .25)$

$= P[(n-X) \geq 6] + 1 - P[(n-X) \geq 1] =$

lost in (n-X)'s? (-3) no (n-X) attempt? (-4)

$= .0544 + 1 - (.9683) = .0544 + .0317 = .0861$

yes-no: not safe to conclude $\quad p > .05$ & "yes"? (-1)

my level: weak evidence, but something.

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2. An IRS auditor randomly selects 3 of 10 income tax returns for a careful audit. Suppose that 4 of the 10 actually contain illegal deductions. Let X denote the number in his sample which contain illegal deductions.

$\boxed{5}$ a. State the name of the distribution of X, and give the values of the parameters.

Hypergeometric, $N=10$, $A=4$, $n=3$

$\boxed{5}$ b. Find $P[X \geq 2]$. $= f(2)+f(3) = \frac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}} + \frac{\binom{4}{3}\binom{6}{0}}{\binom{10}{3}} = \frac{\left(\frac{4 \cdot 3}{2}\right)(6) + 4}{\frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1}} =$

$P[X \geq 2]$? (-2)

$= \frac{40}{120} = \frac{1}{3}$

$\boxed{5}$ c. Find $P[X=3 \mid X \geq 2]$. $= \frac{P[X=3 \ \& \ X \geq 2]}{P[X \geq 2]} = \frac{P[X=3]}{P[X \geq 2]} = \frac{\frac{4}{120}}{\frac{1}{3}} = \frac{12}{120} = \frac{1}{10}$

(-3)

$P[X=3]$ only; (-5)

$\boxed{5}$ d. Find $P[X=3 \mid X \leq 2]$. 0 since disjoint.

$P[X=3 \ \& \ X \leq 2] = P[X=3] + P[X \leq 2]$? (-4)

$P[X=3 \ \& \ X \leq 2] = P[X=3]$? (-2)

only say "not possible"? $(-)$

$\boxed{5}$ e. Are the events $\{X=3\}$ and $\{X \leq 2\}$ independent? Why or why not?

Yes? (-5)

NO, disjoint! know $\{X \leq 2\}$ makes big change in $P[X=3]$.

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3. An inspector on the Alaska Pipeline is responsible for the maintenance of pumping stations. Each station is susceptible to two kinds of failure: pump failure, and leakage. At present pump failures occur only half as often as leakage. The probability of a simultaneous occurrence of both pump failure and leakage is 0.02. $P[P] = \frac{1}{2}P[L]$ $P[P \,\&\, L] = .02$

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a. If a pump station is 72% resistant to failures (without regard to type of failure), what is the probability of a pump failure.

$.28 = 1 - .72 = P[P \text{ or } L] = P[P] + P[L] - P[P \,\&\, L] = P[P] + 2P[P] - .02$

not 1−?

or $.3 = 3P[P]$ $P[P] = \boxed{.1}$

(−3)

$P[L] = 2P[P] = .2$

$\frac{1}{2} \Leftrightarrow 2$? (−3)

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b. If improved pumps are installed, which only fail one fourth as often as leaks occur (but the simultaneous failure probability is still 0.02), what will be the overall resistance to failure of a pumping station?

(.1)? (−2)

$P[P] = \frac{1}{4}P[L] = \frac{1}{4}(.2) = .05$

$4 \Leftrightarrow \frac{1}{4}$? (−3)

$P[\text{no failure}] = 1 - P[P \text{ or } F] = 1 - (P[P] + P[F] - P[P \,\&\, F]) =$

$= 1 - (.05 + .2 - .02) = 1 - .23 = \boxed{.77}$

.23? (−3)

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4. A physical therapist knows that 30% of football games will be played on artificial turf next season. He also knows that a football player's chances of incurring a knee injury are 25% higher if he is playing on artificial turf instead of grass. If a player's probability of knee injury on artificial turf is 0.5, what is the probability that:

$P[A] = .3 \quad P[G] = .7 \quad P[I|A] = .5$

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a. A randomly selected football player incurs a knee injury?

$P[I|A] = \frac{5}{4} P[I|G] =$

$P[I|G] = \frac{4}{5} P[I|A] = \frac{4}{5}(.5) = \frac{2}{5} = .4$

messed up simply? but got -4

$P[I] = P[I|G]P[G] + P[I|A]P[A] = (.4)(.7) + (.5)(.3) = .28 + .15$

$= \boxed{.43}$

+ ard part? -3

got the, but number on? -10

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b. A randomly selected football player with a knee injury incurred the injury playing on grass?

$P[G|I] = \frac{P[G \cap I]}{P[I]} = \frac{P[I|G]P[G]}{P[I]} = \frac{.28}{.43} \approx \boxed{.651}$

This much, but wrong after? -10

P[G]? -7

(.5)x(.3)? -2

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