STOR 155, Section 1, Midterm I
Tuesday, February 24, 2009

Name: ____________________Solution___________________________

Pledge: I have neither given nor received aid on this examination.

Signature: ____

Instructions: Do not do any actual numerical calculations. Answers in a form that you would type into an Excel field, such as "=28*SQRT(82)^2", with a *working* answer, are expected.

1. The workforce in a city has the following breakdown of workers: 10% with a High School education, 70% with High School, but no College, 20% with a College Education. Past experience indicates that workers can perform a given task with success rates: 20% for no H.S., 40% for H.S. no C., 80% for C, education. Find the probability that a randomly chosen worker:

    a. Can perform the given task.

[5]

P[Perf] = P[(Perf & no HS) or (Perf & HS no C) or (Perf & C)]
= P[Perf & no HS] + P[Perf & HS no C] + P[Perf & C]
= P[Perf | no HS] P[no HS] + P[Perf | HS no C] P[HS no C] + P[Perf | C] P[C]
= (0.2 * 0.1) + (0.4 * 0.7) + (0.8 * 0.2)

    b. Is College educated if (s)he can perform the given task.

[5]

Use above, or Bayes Rule:
P[C | Perf] = P[Perf | C] * P[C] / (answer to a above)
= P[Perf | C] * P[C] /
(P[Perf | no HS] P[no HS] + P[Perf | HS no C] P[HS no C] + P[Perf | C] P[C])
= (0.8 * 0.2) / ((0.2 * 0.1) + (0.4 * 0.7) + (0.8 * 0.2))

2. Our factory buys sheet metal from supplier A, and our long run experience has been that 90% of it meets our specifications. Supplier B claims to have a higher level of quality, so we test 30 of his shipments (that have been randomly sampled), and see that 29 of them meet our specifications. Is it safe to conclude that sheet metal from Supplier B is indeed of higher quality?

a. Formulate suitable quantitative hypotheses.

[5]

Let p = Prob. Shipment from B meets specs

H0: $p \leq 0.9$ H1: $p > 0.9$

b. Calculate a p-value to address this question.

[5]

P[X = 29 or m. c. | H0 – H1 boundary]

= $P[X \geq 29 \mid p = 0.9]$

= $1 - P[X \leq 28 \mid p = 0.9]$

= 1 – BINONDIST(28,30,0.9,true)

c. If the numerical answer to part b had been p-value = 0.0051, what would the yes-no conclusion be?

[5]

p-value < 0.05, so safe to conclude B makes sheet metal of higher quality

d. If the numerical answer to part b had been p-value = 0.0051, give a gray level interpretation of the result (in 4 words or less).

[5]

Very strong evidence

3. Are the following assignments of probabilities to outcomes legitimate? If not, why not?

a. Roll a die: P[1] = P[2] = P[3] = P[4] = p[5] = p[6] = ¼.

[5]

No, sum of probs = 1.5, needs to be one.

b. Draw a card: P[diamond] = 12/52, P[club] = 13/52, P[heart]] = 13/52, P[spade] = 14/52.

[5]

Yes, sum of probs = 52/52 = 1. So legitimate.

4. When undergraduates write a 250 word essay (without spell checking,), the number X of errors has the following distribution:

Value of X	0	1	2	3	4
Probability	0.1	0.2	0.3	0.3	0.1

a. Write the event of "at least two errors" in terms of X, and give its probability.

[5]

$\{X \geq 2\}$

$P[X \geq 2] = f(2) + f(3) + f(4) = 0.3 + 0.3 + 0.1$

b. Describe the event $\{X \leq 3\}$ in words, and give its probability.

[5]

"no more than 3 errors"

$P[X \leq 3] = f(0) + f(1) + f(2) + f(3) = 0.1 + 0.2 + 0.3 + 0.3$

c. If it is known that there is an error, what is the chance of 4 errors?

[5]

$$\begin{aligned} P[X = 4 \mid X > 0] &= P[X = 4 \;\&\; X > 0] \,/\, P[X > 0] \\ &= P[X = 4] \,/\, (1 - P[X = 0]) \\ &= 0.1 \,/\, (1 - 0.1) \end{aligned}$$

d. Find $P[X \leq 1 \text{ or } X > 3]$.

[5]

$= P[X \leq 1] + P[X > 3] - 0$

$= f(0) + f(1) + f(4)$

$= 0.1 + 0.2 + 0.1$

e. Find $P\{X \leq 1 \text{ and } X > 3\}$.

[5]

0, this cannot happen

f. Find $P\{X \leq 1 \mid X > 3\}$.

[5]

0, if know $X > 3$, then can't have $X \leq 1$

5. A prisoner rolls a die 3000 times, and observes that the face (out of 6 possible faces) labeled "1" comes up 482 times. Is it safe to conclude that the die is unbalanced?

   a. Formulate suitable hypotheses.

[5]

   Let p = Prob of a 1
   H0: $p = 1/6$ H1: $p \neq 1/6$

   b. Give a P-value for testing these hypotheses.

[5]

   P-value = $P[X = 482 \text{ or m.c.} \mid p = 1/6]$
   $= P[X \leq 482 \text{ or } X \geq 518 \mid p = 1/6]$
   $= P[X \leq 482 \mid p = 1/6] + (1 - P[X \leq 517 \mid p = 1/6])$
   = BINONDIST(482,3000,1/6,true) +
   (1 - BINONDIST(517,3000,1/6,true))

   c. If the numerical answer to b is: p-value = 0.51, give the yes-no result, and a gray level conclusion (4 words or less).

[5]

   Not safe to conclude unbalanced. Evidence is very weak.

6. About 40% of male internet users, ages 18-28 visit an online auction site, such as eBay once a week. In a survey of 25 such users,

   a. What is the exact distribution of the number that did not visit such a site last week?

[5]

   P[not visiting] = 1 – P[visit] = 1 – 0.4 = 0.6
   So Binomial(25,0.6)

   b. What is the probability that exactly 16 of the folks in the study had not visited last week?

[5]

   $P[X = 16]$ = BINOMDIST(16,25,0.6,false)

   c. What is the probability that that at least 14 had not visited?

[5]

   $P[X \geq 14] = 1 - P[X < 14] = 1 - P[X \leq 13]$
   = 1 - BINOMDIST(13,25,0.6,true)