4.14

(a) P(x<=3) = p(1) + p(3) = .1 +.2 = .3
(b) P(x<3) = p(1) = .1
(c) P(x=7) = .2
(d) P(x>=5) = p(5) + p(7) + p(9) = .4+.2+.1 = .7
(e) P(x>2) = p(3) + p(5) + p(7) + p(9) = .2+.4+.2+.1 = .9
(f) P(3<=x<=9) = p(3) + p(5) + p(7) + p(9) = .2+.4+.2+.1 =.9
(g) P(x=3 | x<7) = P(x=3 and x<7) / P(x<7)
= P(x=3) / P(x<7) = .2 / (.1+.2+.4) = .286
(h) P(x>=3 | x<7) = P(3<=x<7) / P(x<7) = (.2+.4) / (.1+.2+.4) = .857
(i) P(x=9 | x<7) = P(x=9 and x<7) / P(x<7) = 0 / P(x<7) = 0

4.12

(a) This is not a valid distribution because \sum p(x) = .9 \= 1.
(b) This is a valid distribution because 0<=p(x)<=1 for all values of x and \sum p(x) = 1.
(c) This is not a valid distribution because p(4) = -.3 < 0.
(d) The sum of the probabilities over all possible values of the random variable is
\sum p(x) = 1.1 > 1, so this is not a valid probability distribution.

4.16

(a) Yes. For all values of x, 0<=p(x)<=1 and
\sum p(x) =
.01+.02+.03+.05+.08+.09+.11+.13+.12+.10+.08+.06+.05+.03+.02+.01+.01 = 1.00.
(b) P(x=16) = .06.
(c) P(x<=10) = p(5) + p(6) + p(7) + p(8) + p(9) + p(10)
= .01 +.02 + .03 + .05 + .08 + .09 = .28
(d) (d) P(5<=x<=15) = p(5) + p(6) + p(7) + p(8) + p(9) + p(10) + p(11) + p(12) + p(13)
+ p(14) + p(15)
= .01 + 02 + .03 + .05 + .08 + .09 + .11 + .13 + .12 + .10 + .08
= .82

4.48

Define x as the number of components that operate successfully. The random variable x is a binomial random variable (the components operates independently and there are only two possible outcomes) with n=4 and p=.85
P(system fails) = P(x=0) = 0.0005

4.40

(a) P(x=2) = P(x<=2) – P(x<=1) = .167 - .046 = .121 (from Table II, Appendix B)
(b) P(x<=5) = .034
(c) P(x>1) = 1 – P(x<=1) = 1 - .919 = .081
(d) P(x<10) = P(x<=9) = 0
(e) P(x>= 10) = 1 – P(x<=9) = 1 - .002 = .998
(f) P(x=2) = P(x<=2) – P(x<=1) = .206 - .069 = .137

4.42

(a) To show that x is an approximate binomial random variable, we must show:

1. n identical trials. Here there are 10 trials which are essentially identical.
2. Two possible outcomes for each trial. For this problem, the two possible outcomes are: Item registers the wrong sale price and Item registers the right sale price. Let S = item registers wrong sale price and F = item registers right sale price.
3. The probability of success is p which is constant from trial to trial. In this problem, p = P(items registers wrong price). We assume that this is constant from trial to trial.
4. Trials are independent. In this problem, we assume that the result of any one scan is not related to the result of any other scan.
5. x = number of items that register the wrong sale price in 10 trials.

Since the five characteristics hold, x is an approximate binomial random variable.

(b) From the data, $p = 83/235 = .35$

(c) $P(x=2) = .176$

$P(x>=2) = 1 – P(x<2) = 1 – P(x=0) – P(x=1) = 1 - .013 - .072 = .915$

(d) The probability of being overcharged = $51/235 = .22$

Let x = number of items that were overcharged in 10 trials.

$P(x>=2) = 1 – P(x<2) = 1 – P(x=0) – P(x=1)$

$= 1 - .083 - .235 = .682$

4.44

Define x as the number of physically healthy patients that seek medical assistance. The random variable x is a binomial random variable (the patients are independently chosen with two possible outcomes).

(a) When n = 15 and p = .1

$P(x>=5) = 1 – P(x<=4)$

$= 1 - .987$ (Table II, Appendix B)

$= .013$

(b) When n = 15 and p = .4

$P(x>=5) = 1 – P(x<=4)$

$= 1 - .217$ (Table II, Appendix B)

$= .783$

(c) We did find 5 of 15 patients seeking medical assistance when they were physically healthy. In part (a), we found the probability of finding 5 or more was only .013 when p = .10. Since this did occur, p is probably larger than .10.