Statistics 23, Section 1, Midterm I
Tuesday, September 21, 1999

Name: _____Solution_____

Pledge: I have neither given nor received aid on this examination.

Signature: _____

Instructions: Show all work, but do not do hard arithmetic (an answer of the form $\binom{8}{3} \cdot 3^7$ is fine).

1. A company makes 50% of its cars at Factory A, 30% at Factory B and the rest at Factory C. Factory A produces 10% lemons, Factory B produces 15% lemons, and Factory C produces 20% lemons. If a car is randomly selected for the overall production system,

a. What is the probability that it is a lemon?

[8]

$P\{A\} = 0.5,\ P\{B\} = 0.3,\ P\{C\} = 0.2,\ P\{L|A\} = 0.1,\ P\{L|B\} = .15,\ P\{L|C\} = 0.2$

$P\{L\} = P\{(L \text{ and } A) \text{ or } (L \text{ and } B) \text{ or } (L \text{ and } C)\} = P\{L \& A\} + P\{L \& B\} + P\{L \& C\}$

$= P\{L|A\}\ P\{A\} + P\{L|B\}\ P\{B\} + P\{L|C\}\ P\{C\}$

$= (0.1)\ (0.5) + (0.15)\ (0.3) + (0.2)\ (0.2) = 0.05 + 0.045 + 0.04 = 0.135$

b. If it turns out to be a lemon, then what is the probability that it was built at Factory C?

[7]

$P\{C|L\} = P(C \text{ and } L\} / P\{L\} = P\{L|C\}\ P\{C\} / P\{L\} = (0.2)\ (0.2) / 0.135$

Or use Bayes rule:

$$P\{C|L\} = \frac{P\{L|C\}\ P\{C\}}{P\{L|A\}\ P\{A\} + P\{L|B\}\ P\{B\} + P\{L|C\}\ P\{C\}}.$$

$$= \frac{(0.2)\ (0.2)}{(0.1)\ (0.5) + (0.15)\ (0.3) + (0.2)\ (0.2)}.$$

2. The IRS says about 5 in 1000 tax returns with income below $50,000 are audited, as are 10 in 1000 for income between $50,000 and $99,999, and 25 in 1000 for income $100,000 and more. P{A|Lo} = 0.005, P{A|mid} = 0.01, P{A|Hi} = 0.025.

(a) If 10 taxpayers are chosen at random from the high income group, write a (complete) formula that could be used in an Excel formula bar to calculate the probability that: X = # A, X~Bi(10, 0.025).

(i) Either none, or else at least 4 of them get audited.

[5]

P{X = 0 or X >= 4} = P{X = 0} + 1 – P{X <= 3} =

=BINOMDIST(0,10,0.025,FALSE) + 1 - BINOMDIST(3,10,0.025,TRUE)

(ii) Exactly 2 of them gets audited, if it is known that at least 2 are audited.

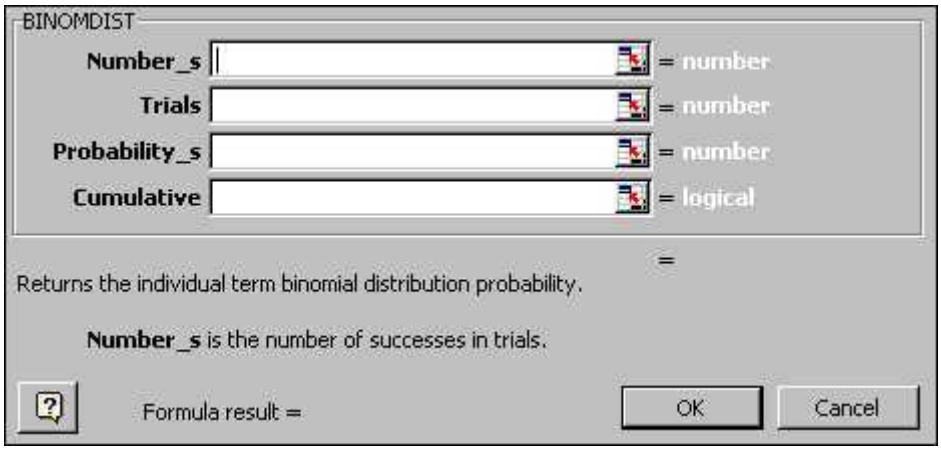
[5]

P{X = 2|X >= 2} = P{(X = 2) and (X >= 2)} / P{X >= 2} = P{X = 2} / P{X >= 2} =

=BINOMDIST(2,10,0.025,FALSE)/(1-BINOMDIST(1,10,0.025,TRUE))

(b) Fill out the Excel menu below to calculate the probability that from a group of 15 taxpayers chosen randomly from the low income group, exactly one of them gets audited.

[5]



1 15 0.005 false

(c) If three taxpayers are randomly selected from the low income groups, and four are selected from the high income group, write a complete Excel formula to calculate the probability that none of these will be audited.

[5]

P{(none Lo) and (none Hi)} = P{none Lo} P{none Hi} =

=BINOMDIST(0,3,0.005,TRUE)*BINOMDIST(0,4,0.025,TRUE)

3. Suppose events A, B and C all have probability 0.5, and A and B are mutually exclusive, and B and C are independent.

a. Find $P\{A \text{ or } B\}$.

[5]

= P{A} + P{B} – P{A and B} = 0.5 + 0.5 – 0 = 1

b. Find $P\{B \text{ or } C\}$.

[5]

= P{B} + P{C} – P{ and C} = P{B} + P{C} – P{B and C} =

= 0.5 + 0.5 – (0.5) (0.5) = 0.75

4. The random variable X has distribution:

x	0	1	2	4
$f(x)$	0.3	0.2	0.1	0.4

a. Find $P\{1 \le X < 4\}$.

[5]

= f(1) + f(2) = 0.2 + 0.1 = 0.3

b. Find $P\{X = 4 \mid X \ge 2\}$.

[5]

= P{(X = 4) and (X >= 2)} / P{X >= 2} = P{X = 4}/ P{X >= 2} =

= f(4) / [f(2) + f(4)] = 0.4 / (0.1 + 0.4) = 0.8

c. Find $P\{X = 4 \mid X \le 2\}$.

[5]

= P{(X = 4) and (X <= 2)} / P{X >= 2} = 0

5 The following table shows the numbers of adults from a small town in age income categories, entered into an Excel spreadsheet, for marketing research purposes.

E6 = =SUM(B6:D6)

	A	B	C	D	E	F
1			Income			
2	Age	<$20K	$20K-50K	>$50K	Totals	
3	<25	5793	3689	2594	12076	
4	25-45	2309	4712	3695	10716	
5	>45	249	938	1219	2406	
6	Totals	8351	9339	7508	25198	
7						

Consider the following events, for a randomly chosen person:

A: {Person is 25 or over }
B: {Person is 45 or under}
C: {Person has income between $20K and $50K}
D: {Person has at least $20K income}

a. Write (complete) Excel formulas that could be entered into a formula bar to calculate:

i. $P\{A \text{ and } B\}$.

[5]

= P{(25 or over} and {45 or under)} = P{25 – 45} =E4/E6

ii. $P\{A \mid B\}$.

[5]

= P{A and B} / P{B} =E4/(E3+E4)

iii. $P\{D \mid A\}$.

[5]

= P{D and A} / P{A} =(C4+D4+c5+D5)/(E4+E5)

iv. $P\{A \text{ or } B\}$.

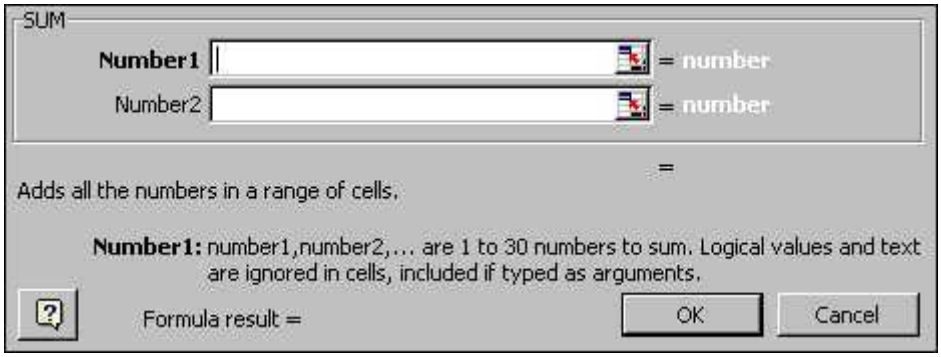
[5]

= P{(25 or over) or (45 or under)} =1

since includes everything

b. One way of finding $P\{A \text{ or } C\}$, is by a sum of table values, divided by the total. Fill out this menu, to calculate the sum needed in the numerator.

[5]



E4:E5 C3

c. Another way to find $P\{A \text{ or } C\}$, is via the "or" rule for probabilities. Write an Excel formula for solving the problem this way.

[5]

= P{A} + P{C} – P{A and C} =((E4+E5+C6-(C4+C5))/E6

d. Describe how you would check whether the events A and D are independent or not.

[5]

Show any one of P{A|D} = P{A}, P{D|A} = P{D}

or P{A and D} = P{A} P{D}.

e. Are the events A and B mutually exclusive? Why or why not?

[5]

No, P{A and B} is not 0.