

Statistics 31, Section 3, Midterm II, **Solution**
 Tuesday, November 14, 2000

Name: _____

Pledge: I have neither given nor received aid on this examination.

Signature: _____

Instructions: Do not do any actual numerical calculations. Answers in a form that you would type into an Excel field, such as “=28*SQRT(82)^2”, with a *working* answer, are expected).

[points per part]

1. A company makes 50% of its cars at Factory A, 30% at Factory B and the rest at Factory C. Factory A produces 10% lemons, Factory B produces 15% lemons and Factory C produces 5% lemons. A car is chosen at random. What is the probability that:

- a. It is a lemon?

[10]

$$P\{A\} = 0.50, \quad P\{B\} = 0.30, \quad P\{C\} = 0.20$$

$$P\{L|A\} = 0.10, \quad P\{L|B\} = 0.15, \quad P\{L|C\} = 0.05$$

$$\begin{aligned} P\{L\} &= P\{(L \text{ and } A) \text{ or } (L \text{ and } B) \text{ or } (L \text{ and } C)\} = \\ &= P\{L \text{ and } A\} + P\{L \text{ and } B\} + P\{L \text{ and } C\} = \\ &= P\{L|A\} P\{A\} + P\{L|B\} P\{B\} + P\{L|C\} P\{C\} = \\ &= 0.10*0.50 + 0.15*0.30 + 0.05*0.20 \end{aligned}$$

- b. It came from Factory B if it is a lemon?

[10]

$$P\{B|L\} = P\{B \text{ and } L\} / P\{L\} = \dots$$

Or Bayes Rule:

$$\begin{aligned} P\{B|L\} &= P\{L|B\} P\{B\} / (P\{L|A\} P\{A\} + P\{L|B\} P\{B\} + P\{L|C\} P\{C\}) \\ &= (0.15*0.30) / (0.10*0.50 + 0.15*0.30 + 0.05*0.20) \end{aligned}$$

[20]

2. The weights of a random sample of 25 runners averaged 60 kg. Suppose that the standard deviation of the population is known to be 10 kg.

a. What is $\sigma_{\bar{X}}$, the standard deviation of the sample average \bar{X} ?

[5]

$$\text{sigma} / \text{sqrt}(n) = 10 / \text{sqrt}(25)$$

b. Find the 99% margin of error for estimating the population mean μ using \bar{X} .

[5]

$$=\text{confidence}(0.01,10,25)$$

c. Give a 90% confidence interval for μ .

[5]

$$\text{left end: } =60-\text{confidence}(0.1,10,25)$$

$$\text{right end: } =60+\text{confidence}(0.1,10,25)$$

d. Exactly how would the confidence interval in (c) change if the sample average were based on a random sample of 100 runners?

[5]

same center, but length would decrease by factor of $\text{sqrt}(4) = 2$

e. How large a sample would be required to estimate μ within ± 0.1 kg with 95% confidence?

[5]

$$=(10*\text{NORMINV}(0.975,0,1)/0.1)^2$$

[25]

3. A household is called prosperous if its income exceeds \$75,000, and called educated if the householder completed college. 20% of all households are prosperous, 30% are educated, and 19% are prosperous and educated. If a household is chosen at random:

a. What is the probability that it either is educated, or else is prosperous?

[5]

$$\begin{aligned} P\{\text{Ed. or Pros.}\} &= P\{\text{Ed.}\} + P\{\text{Pros.}\} - P\{\text{Ed. and Pros.}\} = \\ &= 0.30 + 0.20 - 0.19 \end{aligned}$$

b. What is the probability that it is educated given that it is prosperous?

[5]

$$\begin{aligned} P\{\text{Ed.}|\text{Pros.}\} &= P\{\text{Ed. and Pros.}\} / P\{\text{Pros.}\} = \\ &= 0.19 / 0.20 \end{aligned}$$

c. Is the event that it is educated independent of the event that it is prosperous? Why or why not?

[5]

No, $P\{\text{Ed.}|\text{Pros.}\}$ is not equal to $P\{\text{Ed.}\}$

4. A box label claims that on average boxes contain 40oz. A random sample of 12 boxes shows an average of 39oz., with $s = 2.2$. To see if we should dispute the claim, consider the hypotheses:

$$H_+ : \mu > 40 \quad H_0 : \mu = 40 \quad H_- : \mu < 40$$

a. Find the p-value to assess the strength of the evidence in favor of H_+ .

[10]

$$\begin{aligned} \text{p-val} &= P\{\text{what was seen or more conclusive} | H_0\} = \\ &= P\{\bar{X} > 39 | \mu = 40\} = \\ &= 1 - \text{NORMDIST}(39, 40, 2.2/\text{SQRT}(12), \text{TRUE}) \end{aligned}$$

b. If the p-value to test H_- were equal to 0.0613, interpret the results from both the “yes-no” and the “gray level” viewpoints.

[5]

yes – no: no strong evidence at level 0.05

gray level: evidence is somewhat strong in the direction of the label being wrong.

[30]

5. According to government data, 15% of employed men have never been married.
- a. If 12 employed men are selected at random, what is the probability that at least 10 have never been married?

[5]

let $X = \#$ is sample never been married. $X \sim \text{Bi}(12, 0.15)$

$$\begin{aligned} P\{X \geq 10\} &= 1 - P\{X < 10\} = 1 - P\{X \leq 9\} \\ &= 1 - \text{BINOMDIST}(9, 12, 0.15, \text{TRUE}) \end{aligned}$$

- b. If 12 employed men are selected at random, what is the probability that less than 4 have been married?

[5]

Use X as above, $P\{\text{less than 4 have been}\} = P\{\text{more than 8 never been}\} = P\{X > 8\} =$
 $1 - P\{X \leq 8\} = 1 - \text{BINOMDIST}(8, 12, 0.15, \text{TRUE})$

or let $X = \#$ have been married, $X \sim \text{Bi}(12, 0.85)$
 $P\{X < 4\} = P\{X \leq 3\} = \text{BINOMDIST}(3, 12, 0.85, \text{TRUE})$

- c. 12 employed men are selected at random, what is the mean number that have never been married?

[5]

$$\text{mean} = n p = 12 * 0.15$$

- d. Let X denote number the number who have never been married, in a random sample of 12 employed men. What is the standard deviation of X ?

[5]

$$\text{s.d.} = \sqrt{n p (1 - p)} = \text{SQRT}(12 * 0.15 * (1 - 0.15))$$

- e. If 1200 employed men are selected at random, what is the probability that at least 100 have never been married?

[5]

Note: BINOMDIST gives an error message, so must use normal approx:

$$X \sim N(np, \sqrt{np(1-p)})$$

$$P\{X \geq 100\} = 1 - P\{X \leq 100\} = 1 - \text{NORMDIST}(100, 12*0.15, \sqrt{12*0.15*(1-0.15)}, \text{TRUE})$$

[25]