Significance in Scale Space

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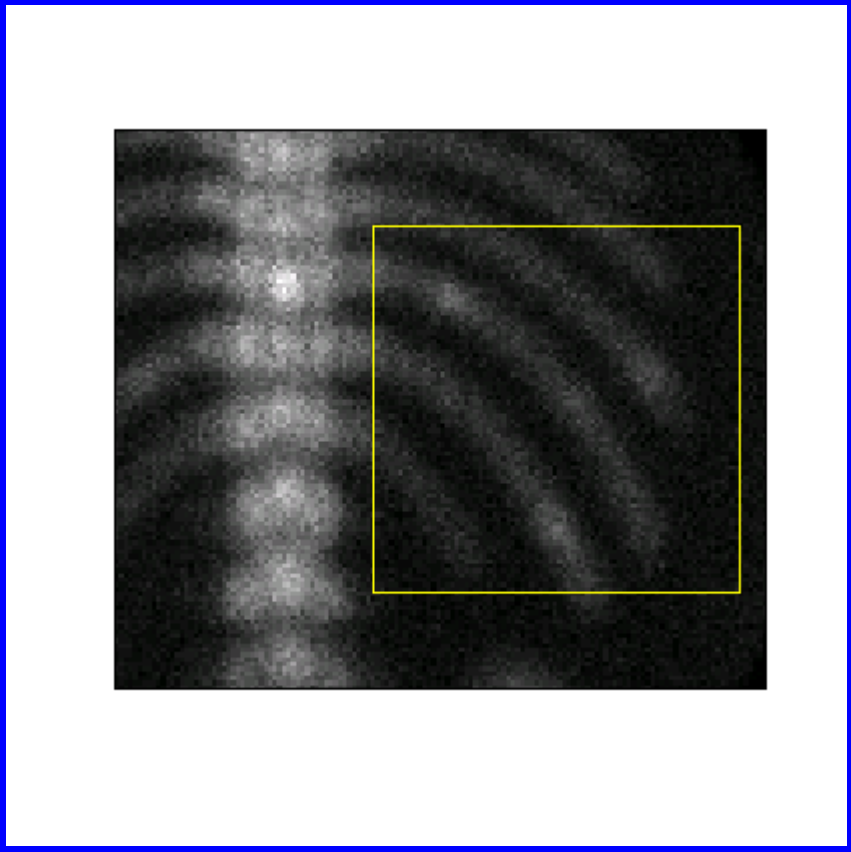
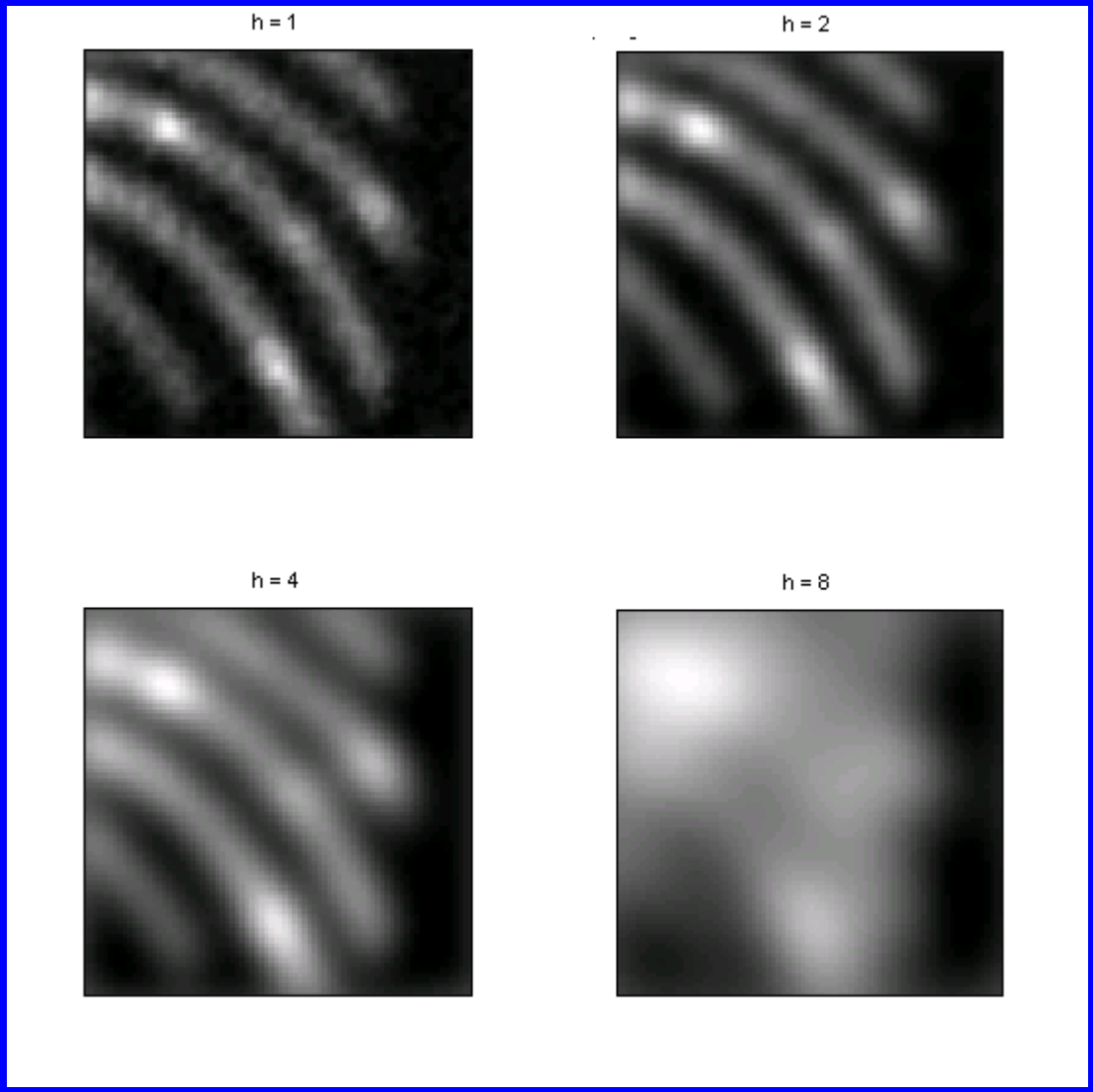
Gamma Camera Phantom Image



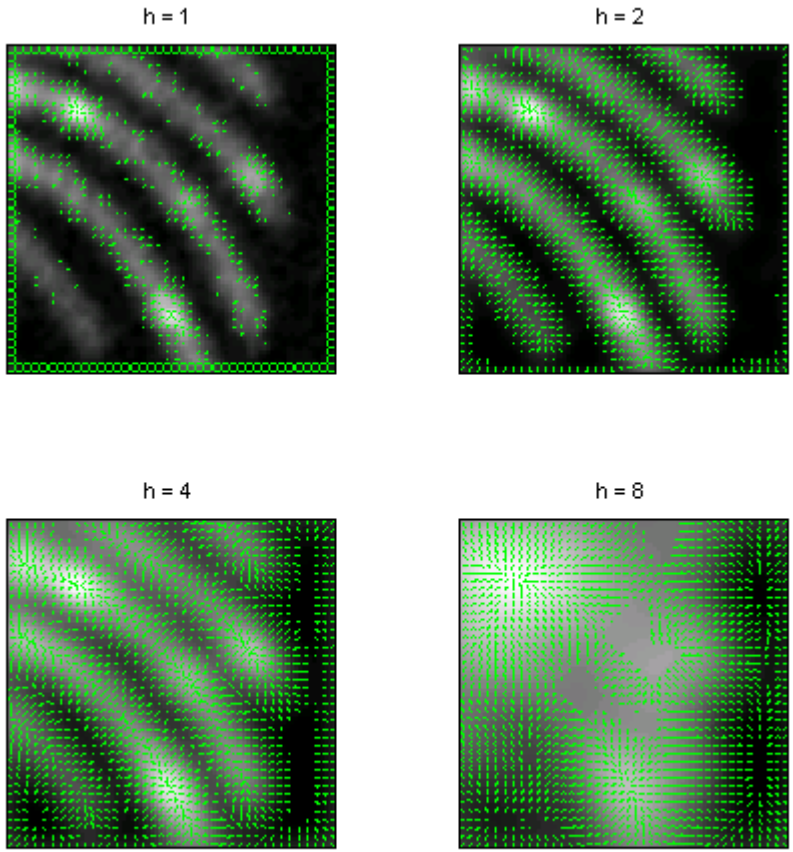
Subimage for later analysis

Gaussian Window Smooths



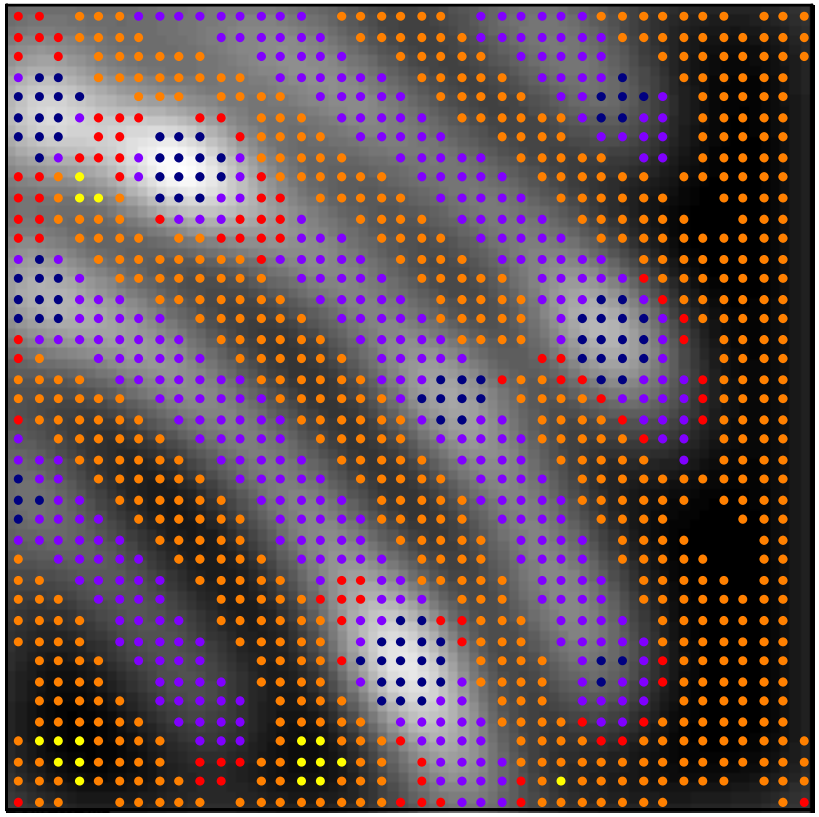
$$\underline{\hat{s}}_h = \underline{K}_h * \underline{Y}$$

Gradient Based S^3

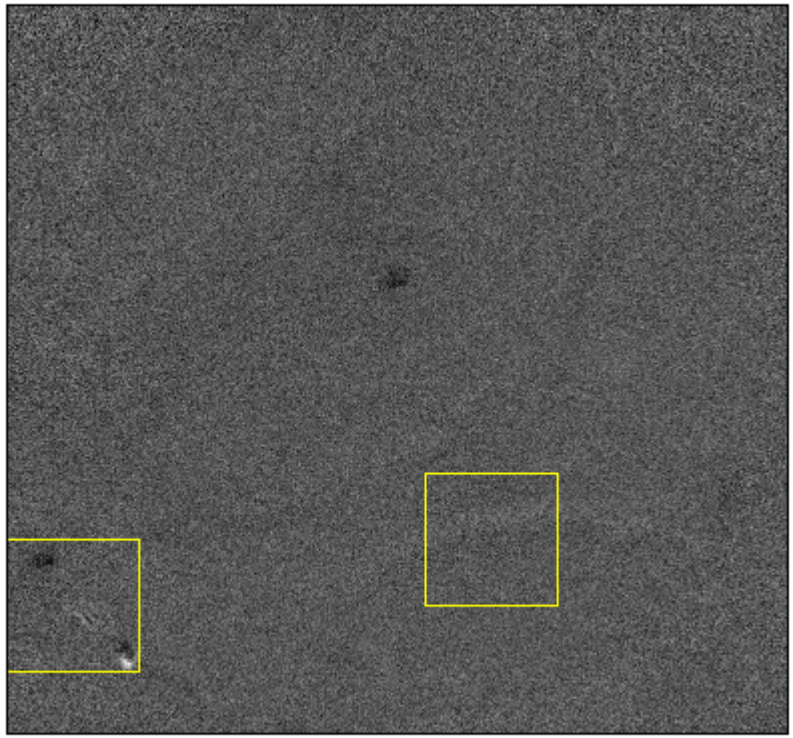


Curvature Based S^3

h = 4



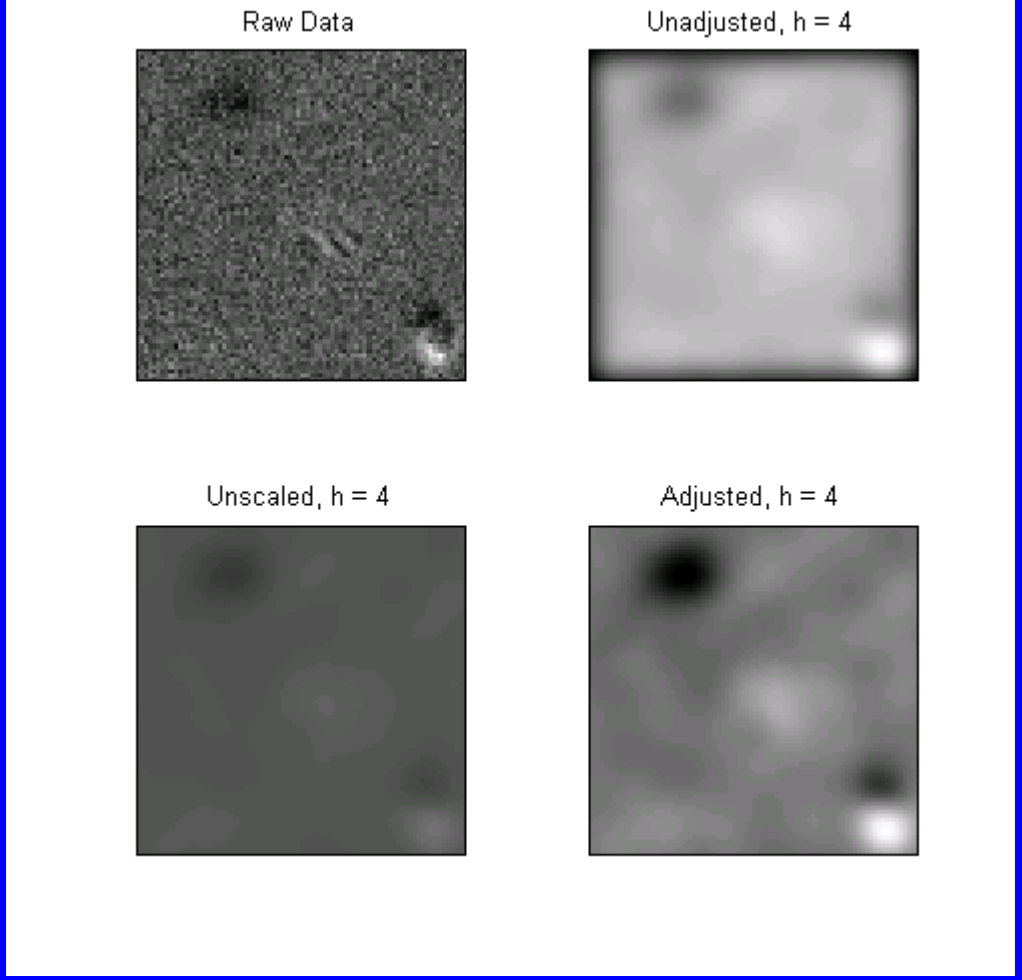
Electrical Activity Image



Subimages for later analysis

Boundary Effect Handling

Electrical Activity Data



$$\underline{\hat{s}}_h = A(\underline{Y}) + \underline{K}_h * [\underline{Y} - A(\underline{Y})]$$

Simultaneous Inference, I

- Via Effective Sample Size:

$$\underline{ESS} = \left(\underline{K}_h * \underline{1}\right) / K_h(0,0)$$

(number of points in kernel window)

- Number of Independent Averages:

$$l = \frac{n \cdot m}{Avg(\underline{ESS})}$$

Simultaneous Inference, II

- Significance Level Adjustment:

$$\alpha = P\{k-\text{th C.I. not covering}, k = 1,\ldots,l\}$$
$$= 1 - P\{\text{C.I. covers}\}^l = 1-(1-\alpha')^l$$

Variance Estimation

- Known Variance (MRI)

- Local Variance Estimate

- Pooled Variance Estimate

Significant Gradient

When gradient “significantly > 0”,

Draw arrow in gradient direction

When arrows too short, use 2 x 2 pixel blocks,

and scale by number significant

Distribution Theory: based on C.L.T. to

Chi-Squared distribution

Significant Curvature

When eigenvalues $\lambda_+ \geq \lambda_-$ of Hessian matrix are "significantly > 0", draw dots as :

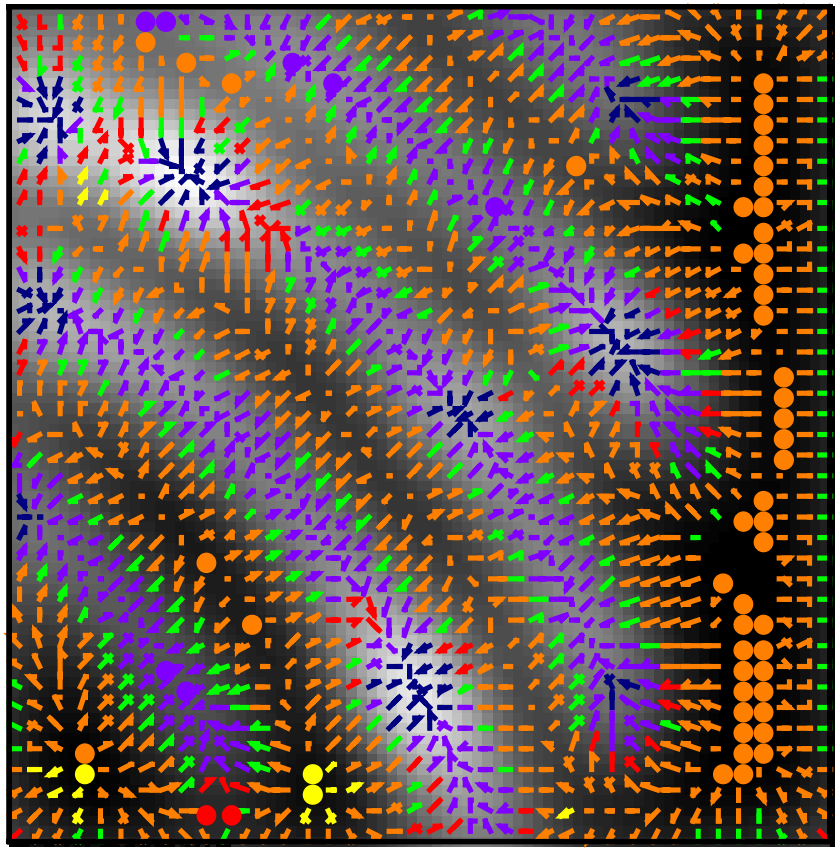
hole	$\lambda_+, \lambda_- > q$
long valley	$\lambda_+ > q, \lvert\lambda_-\rvert < q$
saddle point	$\lambda_+ > q, \lambda_- < -q$
long ridge	$\lambda_+ \rvert < q, \lambda_- < -q$
peak	$\lambda_+, \lambda_- < -q$

(where q is appropriate quantile)

Distribution Theory: based on C.L.T. and simulation

Combining Gradient & Curvature

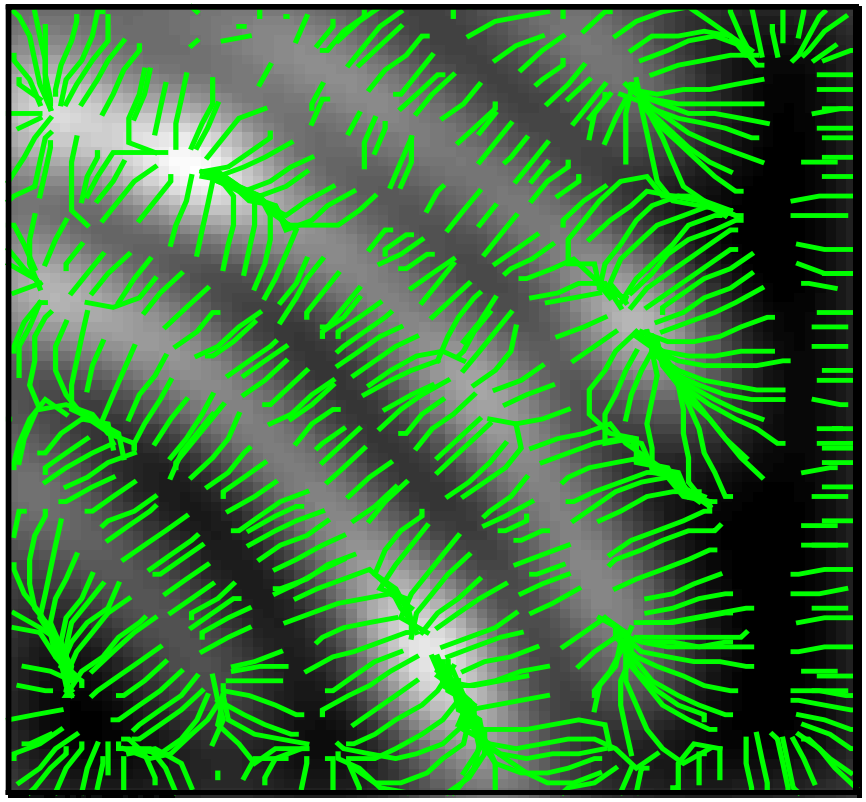
h = 4



Streamlines

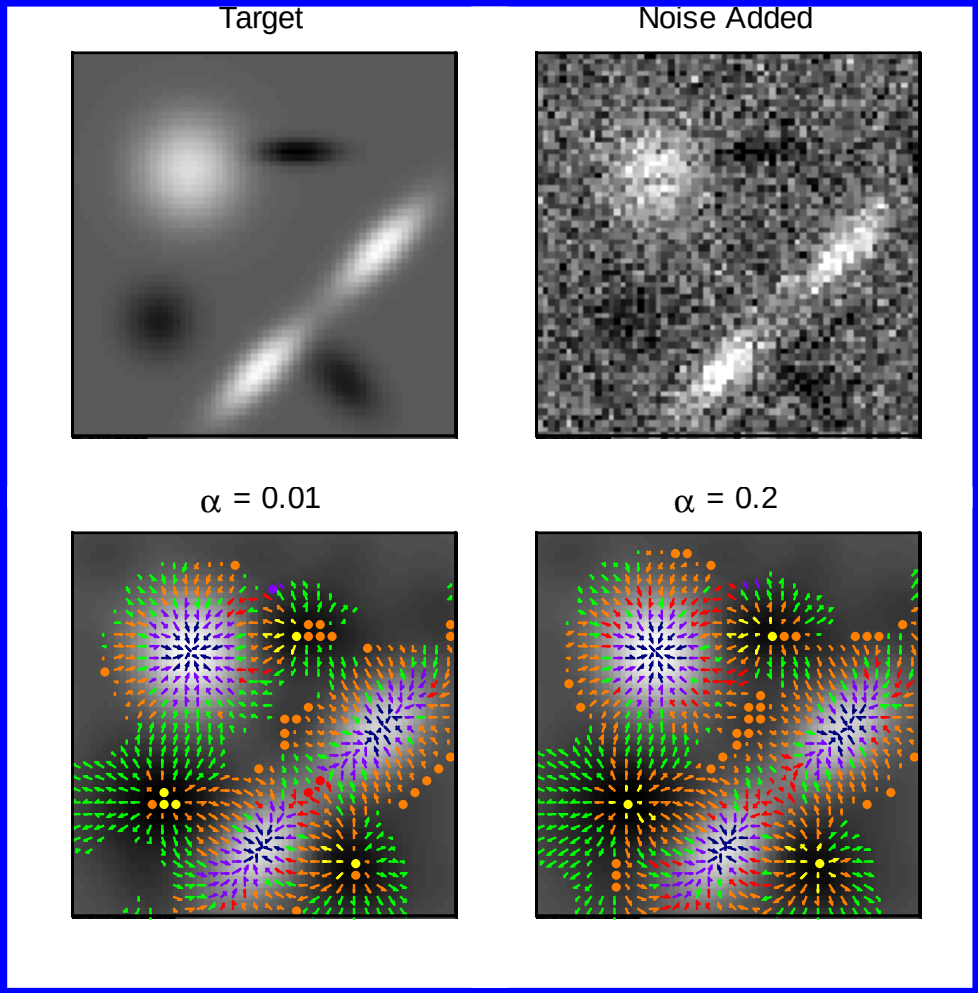
- show gradient directions, where significant

h = 4



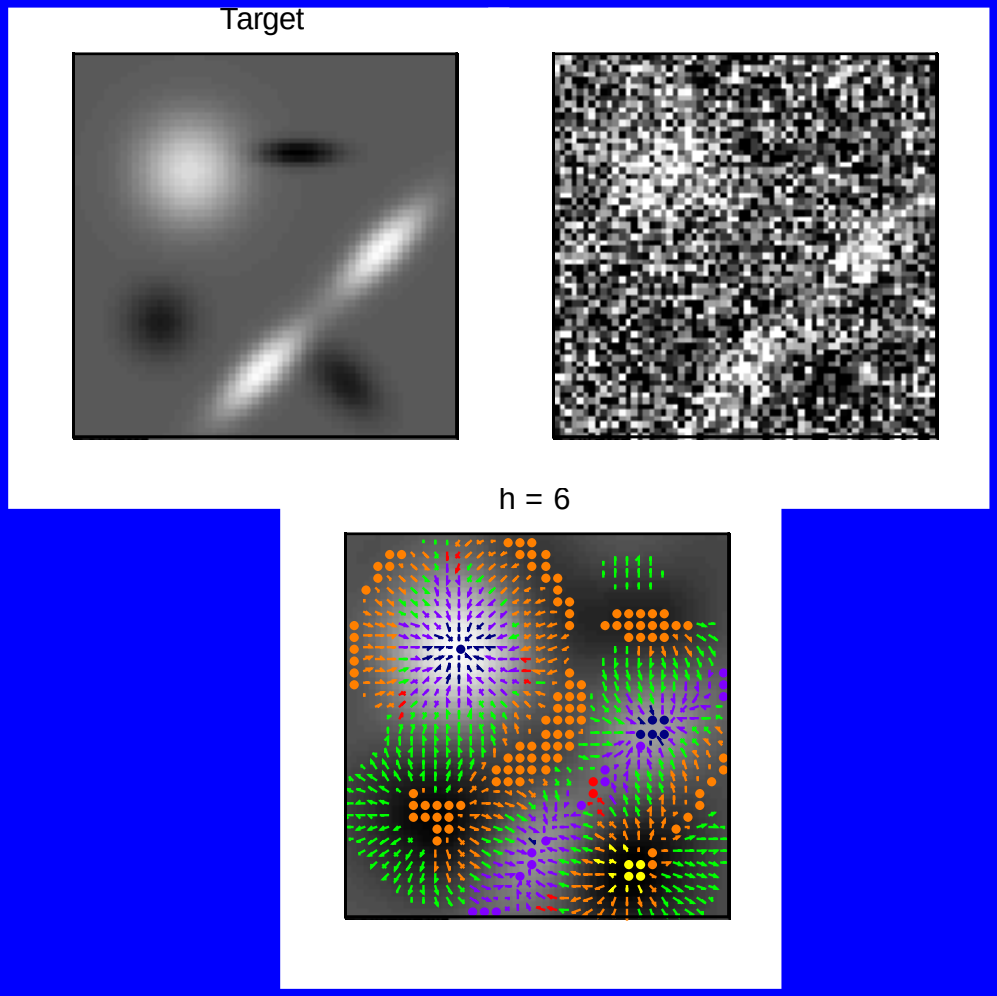
Simulations, I

- Effect of significance level α:



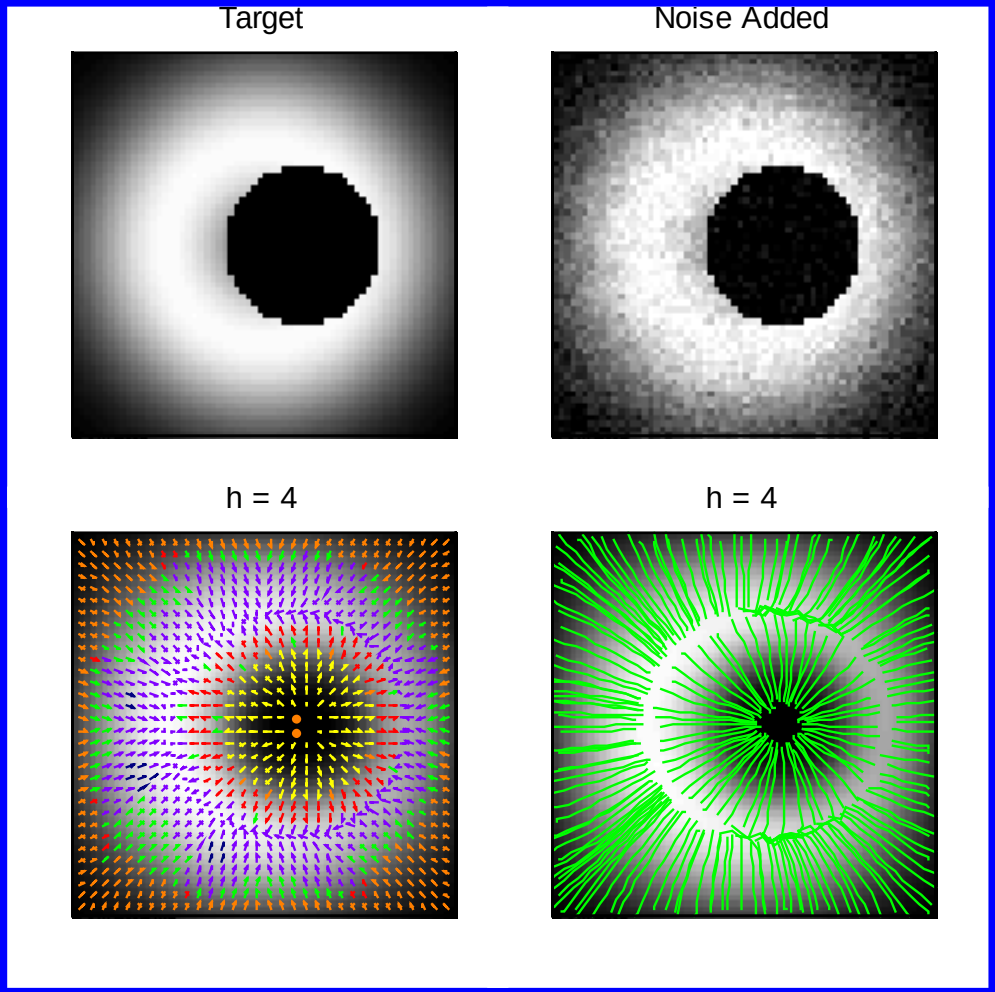
Simulations, II

- High Noise Performance:



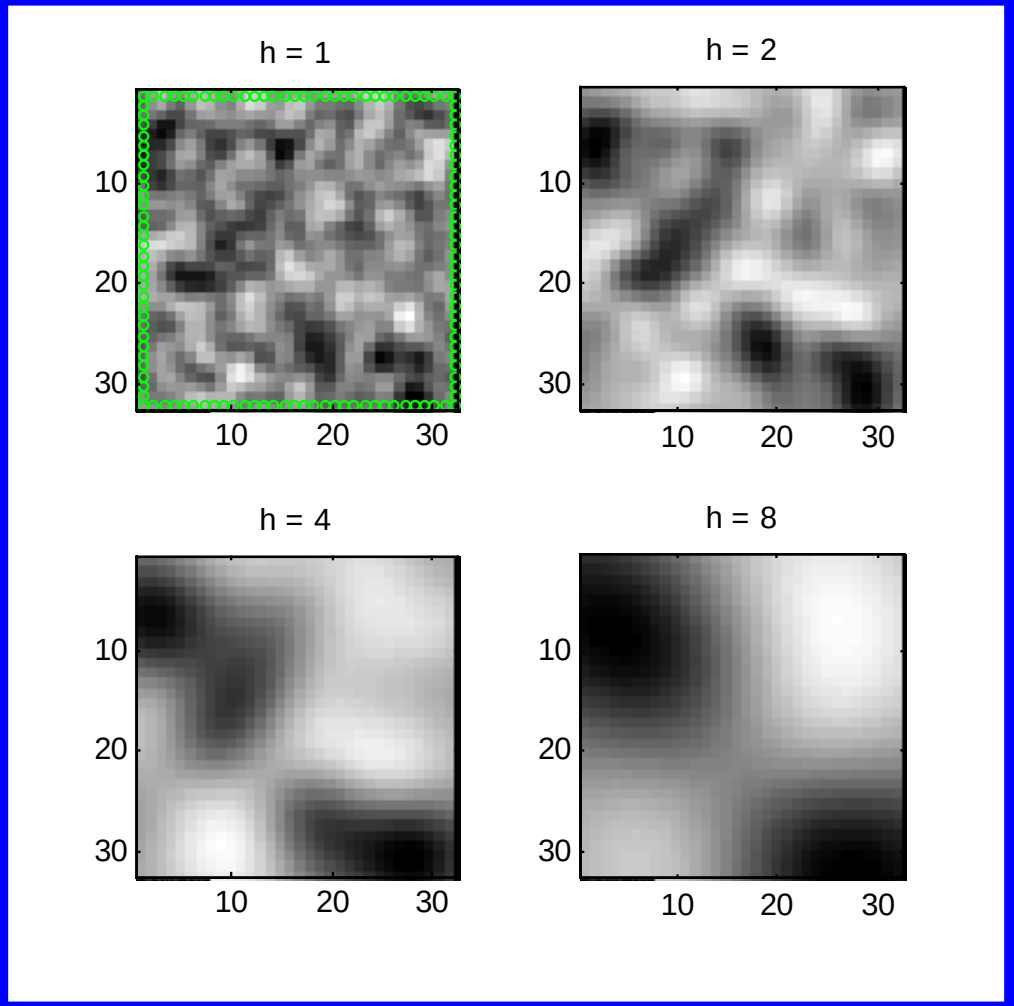
Simulations, III

- Comparison of Methods:

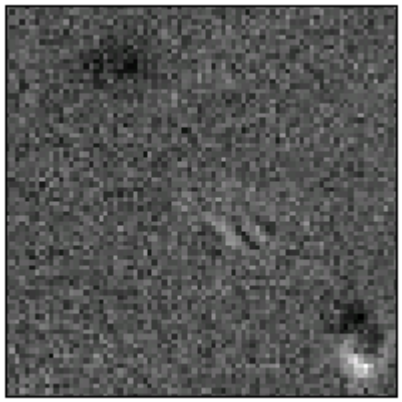


Simulations, IV

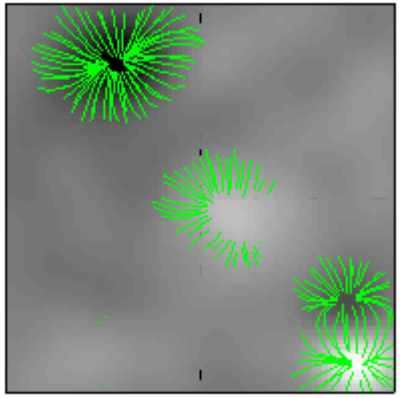
- No Signal:

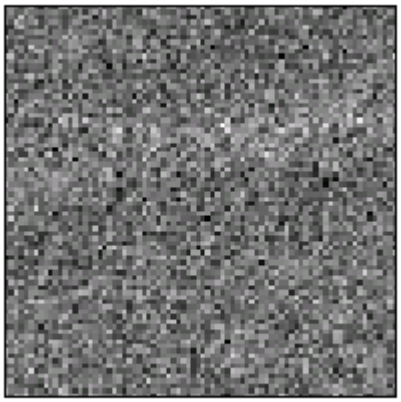


Real Data I, Brain Activity

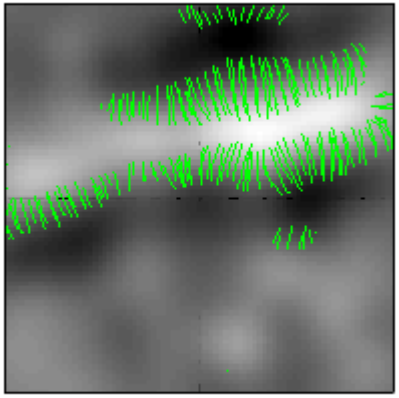


h = 5

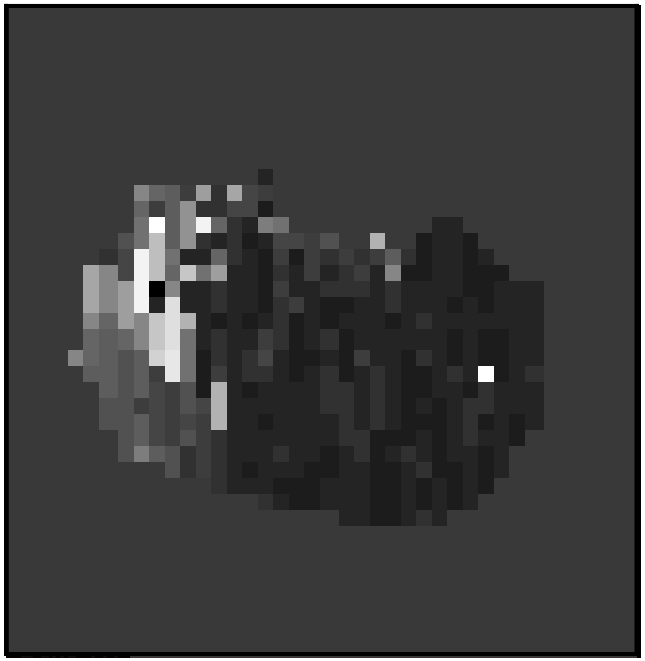




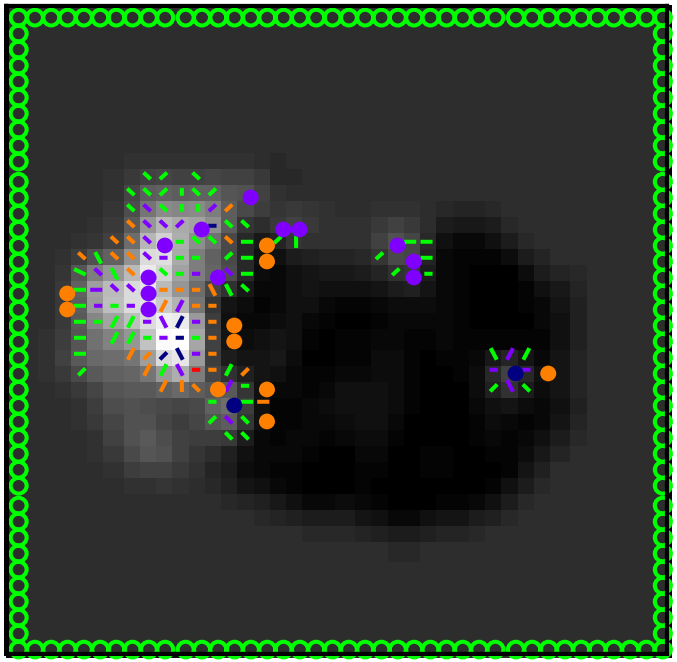
h = 5



Real Data II, MRI Perfusion

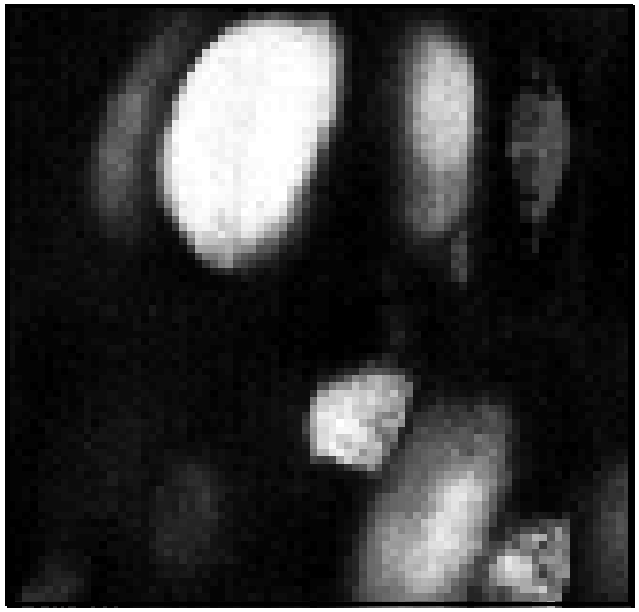


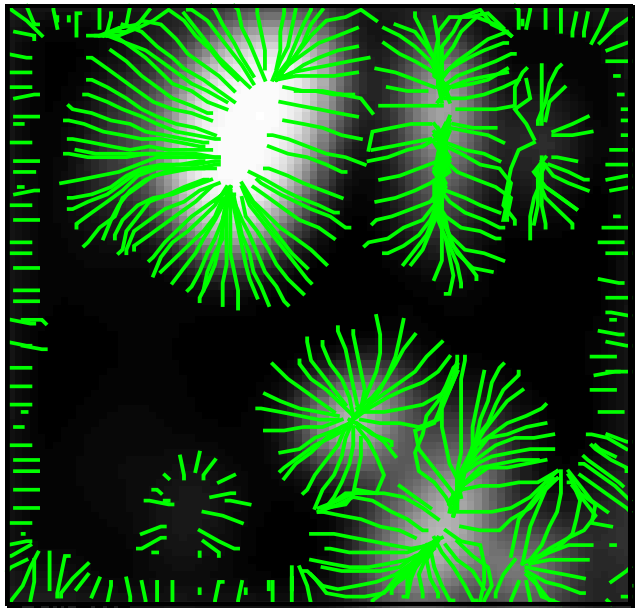
h = 1



Real Data III, Confocal

h = 3





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